Atmospheric Halos and the Search for Angle x

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Walter Tape and Jarmo Moilanen

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Front cover: Halo display with 18°, 23°, and 35° circular halos. Fairbanks, Alaska, January 31, 2005.

Frontispiece: Lunar halo display with 9° and 22° circular halos, 22° and 46° column arcs, and 9°, 22°, and 24° plate arcs. Georg von Neumayer Station, Antarctica, July 11, 1987. Photo © Klaus Sturm.

Back cover: Halo display with M-arc. Viitasaari, Finland, January 8, 1999. Photo © Rainer Vilkkilä.

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Preface

What does an ice crystal look like? You would think that we would know by now. We do indeed know quite a bit about ice crystals, yet there is one intriguing variety that has remained obscure until recently. On occasion crystals of this sort occur in the atmosphere by the billions, so in one sense they are not at all uncommon or exotic. Moreover, they leave a characteristic signature in the sky, and their existence had therefore been conjectured, even in the nineteenth century. Nevertheless, for a long time nobody knew for sure what the crystals looked like. The crystals were seen for the first time only a few decades ago, and they have been seen well only recently. The shape of these crystals is determined by the so-called angle x . The search for angle x is the effort to discover what the crystals look like.

These crystals would not be so intriguing were it not for their signature in the sky. By signature we mean atmospheric halos, something akin to the familiar ring around the moon. But the halos associated with these special crystals are easily recognizable as being different—especially in size—from the ring around the moon; they are “odd radius” halos. For a long time the existence and nature of odd radius halos was as murky as that of the crystals that make them. We now think that we are beginning to understand them, but much remains to be done.

Some of what remains to be done is just a fleshing out of the photographic record of these halos. Here anyone can make a contribution, anyone who has modest camera equipment and is alert for what to watch for. We have no doubt that with increased awareness new halos will be seen and photographed, and we hope that some of you will be the ones to do so.

The ultimate goal of this book is to tell what is known about odd radius halos. But as you read on, you will see that the odd radius halos are partly just an excuse for us to talk about halos in general. One can even argue that it is the odd radius halos that *are* the “halos in general,” because it turns out that it is the odd radius halos that are in a sense typical, and it is the more traditional and more common halos that are exceptional—exceptional in the sense that they have special properties. The general context demanded by odd radius halos is the best setting for thinking about even the common halos.

The study of odd radius halos is very much a work in progress. There are still halo displays that we do not understand very well, and there are halo displays that we do not understand at all. Only recently have we begun to make the accurate measurements of halo radii that are necessary for testing the theory. And only recently have we begun to get electron microscope photographs of the ice crystals that may hold some of the secrets of odd radius halos.

In writing this book, we have tried to present things in such a way that you, the reader, can come to understand how we know what we know. The theory of halos is simple enough so that this can be done, at least to a large extent. So you should not have to take a lot on faith; we have tried to present enough of the reasoning so that you can largely judge the logic of the theory for yourself, and we have presented enough of the data—mainly halo and ice crystal photographs—so that you can test the predictions of the theory yourself. We have omitted some topics that we thought were speculative or whose conclusions were hard to justify in a short space.

The level of difficulty of the book is uneven, we admit. Most of the book can be followed by attentive readers with no special background, but parts of it, such as the treatment of crystallography, require some college level mathematics. Obviously you do not need our permission to pick and choose what to read.

A word about the halo photographs: Most of them are scans of traditional 35mm slides. Nearly all have been enhanced to some degree using digital unsharp masking. With a few exceptions, the resulting images still show less than was visible in the halo displays themselves. The main exceptions are the lunar halo displays, where, due to the long time exposures required, a slide will sometimes reveal more than was seen at the time of the display. But never have we added anything artificially to the halo photographs.

The degree of enhancement varies from one photo to another throughout the book, but on a given photo the enhancement has been applied uniformly over the entire image, rather than to selected portions, and you will be able to estimate the degree of enhancement by looking for vignetting at what would normally be well-defined edges. In Figure 18.5, for example, the prominent white vignetting of the sun-blocking stick is a sign of strong enhancement.

Well developed halos—and not just the odd radius halos—are beautiful, and so are the atmospheric ice crystals that cause the halos. We think that the science that links the crystals with the halos is beautiful in its own right. We hope you will agree.

Eva Bliembach, Les Cowley, John Hallett, Claudia Hinz, Dixon Jones, Gunther Können, Marko Riikonen, Jukka Ruoskanen, and Eva Seidenfaden all helped with the book, whether they knew it or not, and we want to thank them. We also want to thank everyone who contributed photographs. Special thanks are due our colleague Ken Severin at the Advanced Instrumentation Laboratory, University of Alaska Fairbanks, who contributed the crystal photographs taken with the electron microscope.

This work was supported in part by National Science Foundation grant OPP-9419235. We thank the National Science Foundation for giving us the opportunity to study halos and ice crystals in Antarctica.

Introduction

Most people have seen a ring around the moon. Fewer notice its daytime counterpart around the sun, even though it is common. And few indeed have ever seen a display of concentric rings like that witnessed by Helmut Weickmann, observing from the open cockpit of his Henschel reconnaissance plane over Germany in 1940. Weickmann’s diagram of the display is reproduced in Figure 1.1, and the following is an excerpt from his notes on the display, written as his plane climbed through thin clouds. The 22° halo that he refers to is just the daytime version of the ring around the moon mentioned above.

5000 meters. Haze in the south, probably representing the lower boundary of the cirrostratus, intense 22° halo is visible in it, becoming increasingly clear.

6300 meters. ...cirrostratus with two more concentric rings round the sun.

7700 meters. The halo phenomenon has increased still more by three more rings. All the rings are clearly red inside and bluish to white at the outside. (Weickmann [86, page 67])

FIGURE 1.1 Weickmann’s diagram of his odd radius halo display in 1940. Each semicircle represents a halo. The numbers are Weickmann’s estimates of the angular distances of the halos from the sun. (From Steinmetz and Weickmann [72])

FIGURE 1.2 The 22° halo. With your arm extended and fingers spread, the angular distance from thumb to pinky is about 22° , the same as the angular radius of the halo.

Weickmann was seeing atmospheric halos. The numbers on the rings in his diagram are his estimates of the angular radii of the halos. To get some feeling for these angular distances, you can extend your arm completely and spread your fingers; the angular distance from thumb to pinky finger will be very nearly 22° . The angular radius of the 22° halo is also 22° , as you might guess. So if you cover the sun with the tip of your thumb—still with the arm extended and fingers spread—then when the 22° halo is present, the tip of your pinky finger will be on the halo (Figure 1.2).

You will find, once you begin to look for halos, that they are surprisingly common. But you will also find that when you see a circular halo and measure its angular radius, the halo almost always turns out to be the 22° halo; other circular halos are more or less rare. Nevertheless, when the 22° halo is bright and uniform, try looking for a much larger circular halo. If successful, you are probably seeing the 46° halo. Its angular radius is 46° , more than twice that of the 22° halo. The 22° halo is itself big, but the 46° halo is enormous. Its size and its typically low intensity make it easy to miss, but dedicated observers might see the 46° halo a handful of times during the year. Probably the largest halo in Weickmann’s diagram was the 46° halo.

Halos are due to ice crystals in the atmosphere. The crystals that make good halos are usually shaped like tiny hexagonal prisms. Such crystals can make a 22° halo and a 46° halo, and they can therefore account for two of the six halos that Weickmann reported. But they cannot account for the other four, and this is what makes Weickmann’s observation special. These four halos are the wrong size, that is, their radii are neither 22° nor 46° . They are “odd radius” halos.

The main objective of this book is to tell the story of odd radius halos. There are many of them, not just the circular halos like those that Weickmann saw. Weickmann himself, together with his colleague Hermann Steinmetz, got the story right, according to current thinking, and much of our book will be an elaboration

of their basic explanation. But before tackling odd radius halos we will want to tell some of the fascinating story of halos in general and of the ice crystals that cause them. We will see that there is a rich variety of halos, and that at times they can be stunningly beautiful. The beauty in the halos is very much a manifestation of the beauty in the ice crystals that cause them.

Why should anyone care about halos, and why especially should anyone care about odd radius halos? In general, halos have something to tell about the atmosphere where they form. The fact that halos are better in Antarctica than elsewhere suggests that there is something special about the Antarctic atmosphere. (What it is, we do not know.) Similarly, any halos on Saturn's moon Titan would provide some clues about the composition of the Titan atmosphere. It was for precisely this reason that the Huygens probe looked for halos as it descended into Titan's atmosphere in January of 2005.¹

But such practicalities are not what drives most halo watchers, and they are not what drives us. So instead of practicalities we offer you aesthetics, and we offer you the excitement of the chase. We invite you to watch for these lovely phenomena and, if you are so inclined, to record and photograph what you see. The field of halos is an area of science where anyone has a chance to contribute. Nobody knows where the next great halo display will occur.

Four common halos

Let's begin with four common halos: the 22° halo, the parhelia, the circumzenith arc, and the tangent arc.

The 22° halo We said that the 22° halo is common. In many parts of the world it is visible about 100 days per year, either at night around the moon or in the daytime around the sun. If you have not seen the daytime version, it is probably because you have not looked for it. But if you have looked for it and still not seen it, it may be because you are looking too close to the sun. In the sky the angular radius of 22° feels big, somehow much bigger than what the spread finger method suggests when you apply it sitting in your armchair indoors. So the 22° halo is really nowhere near the sun. When you eventually see it and try to photograph it with a standard camera lens, you will find that the lens is not up to the job, and that it captures only a part of the halo. Most of the halo photographs in this book were taken with wide angle lenses. Such lenses compress everything, including the halos, thus making the halos look unrealistically small.

Sometimes the 22° halo is bright, sometimes it is barely discernible. It is usually

¹At the time of this writing, no halos had been detected in the Huygens data, but the data were still being analyzed. Können [41] has speculated on the sorts of halos that might occur on Titan.

FIGURE 1.3 Parhelia at left and right and circumzenith arc at top. Like most of the photographs in this book, this photograph was taken with a wide angle lens, which compresses the halos and here makes the circumzenith arc look too low. The arc is in reality nearly overhead. South Pole, December 10, 1998.

FIGURE 1.4 Computer simulation of a rather ordinary halo display. Of the halos shown here, only the 46° halo is at all uncommon. **S** = sun, **PH** = parhelia, **CZ** = circumzenith arc, **T** = tangent arc.

colored, but rarely are the colors pronounced. Most often the halo will be colored a rather nondescript orange-brown on the inside and then white on the outside, with at most a suggestion of other colors in between.

Parhelia The parhelia, or sundogs, are almost as common as the 22° halo. They appear as two patches of light to the left and right of the sun (Figure 1.3). Their angular distance from the sun is 22° or a bit more, so when the 22° halo and parhelia occur simultaneously, the parhelia will be on or just outside the 22° halo. In fact, the parhelia often appear simply as enhancements of the 22° halo at left and right. Some parhelia show rather good spectral colors, with red on the inside—the sun side—and blue on the outside. Other parhelia can be entirely white, especially when the sun is high.

Circumzenith arc The next time that you see a parhelion, look at the sky high overhead. If the sun elevation is right, say 15° to 25° , and if the cloud that is making your parhelion extends to the neighborhood of the zenith, you are apt to see the circumzenith arc as well (Figure 1.3). It can be glorious—bright with spectacular spectral colors—but few people outside the halo community ever see it, not because the arc is rare, but because people rarely look up. We predict that you will see the circumzenith arc about 25 days per year if you watch for it.

Tangent arc Another common halo is the tangent arc. When accompanying the 22° halo, the tangent arc is outside the 22° halo but tangent to it on top and bottom. When the arc is well defined, it has a distinctive and lovely shape that changes dramatically with sun elevation. More often, though, the tangent arc will appear only as a vaguely defined region of illumination at the top or bottom of the 22° halo.

Although the tangent arc is technically a single halo, at low to moderate sun elevations it consists of two separate components known as the upper and lower tangent arcs. For low sun the lower tangent arc is of course below the horizon, and so what is normally seen then is just the upper tangent arc. At moderate and higher sun elevations the upper and lower tangent arcs do indeed merge and form one halo, the so-called circumscribed halo.

Figure 1.4 is a computer simulation of a halo display consisting mainly of the above four halos—the 22° halo, the parhelia, the circumzenith arc, and the tangent arc. The display is a bit better than average, but it is far from being a great display. To begin to appreciate how a better halo display might arise, you need to know something about the falling modes of ice crystals.

The way the crystals fall

We said that the crystals that make halos are usually in the shape of hexagonal prisms. These prisms can range from short and squat to long and thin; the former

FIGURE 1.5 (Hexagonal) plate and column crystals. Each crystal has two basal faces and six prism faces

FIGURE 1.6 (Left) Oriented plate crystal. The basal faces of the crystal are horizontal, but there are no other constraints on the crystal. (Right) Oriented column crystal. The crystal axis is horizontal, but there are no other constraints.

are called (hexagonal) *plates*, the latter are (hexagonal) *columns* (Figure 1.5).

The shapes and sizes of crystals influence the way that the crystals orient themselves as they fall through the air. Plate crystals often tend to fall with their basal faces horizontal; such crystal orientations are called *plate orientations*, and the crystals are called oriented plates. Column crystals often tend to fall with their axes horizontal; these orientations are *column orientations*, and the crystals are oriented columns. Oriented plates and oriented columns are falling so as to maximize air resistance. Sometimes, however, air resistance seems to produce no special orientations, and the crystals fall with more or less random orientations. Plate orientations, column orientations, and random orientations are the three common falling modes for ice crystals.

It is not obvious that the crystals should tend to fall in these three ways. Nineteenth century scientists, in fact, had the plate and column falling modes reversed, with the crystals orienting so as to minimize air resistance and thus knifing through the air as they fell. We still do not understand in detail the relation between crystal size and shape on the one hand, and falling mode on the other. In an effort to do so, people have tossed nickels into swimming pools and watched them sink, and they have dropped large styrofoam crystal models from rooftops. Other people have approached the problem more theoretically. But the truth is that nobody knows for sure how a tiny, nearly invisible ice crystal will fall. Some of the most convincing evidence for the existence of the three classes of crystal orientations—plate orientations, column orientations, and random orientations—comes from halo observations, as we will see later.

The existence of different crystal orientation classes makes for a rich variety of halos, because each orientation class gives rise to its own characteristic halos. The parhelia and the circumzenith arc arise in oriented plate crystals, the tangent arc

FIGURE 1.7 Computer simulation of a good halo display. The instructions given to the computer to make this simulation were the same as those for the simulation in Figure 1.4 except that here the tilts of the crystals were made much smaller. (The tilts of the randomly oriented crystals were not changed.) Smaller tilts result in more and better halos. **S** = sun, **PH** = parhelion, **CZ** = circumzenith arc, **T** = tangent arc, **INF** = infralateral arc, **SUP** = supralateral arc, **W** = Wegener arc, **PHC** = parhelic circle, **H₂** = subhelic arc.

FIGURE 1.8 Halo display, South Pole, January 2, 1990.

arises in oriented column crystals, and the 22° halo arises in randomly oriented crystals.

Crystal orientations are of course never perfect. Oriented plates and columns, rather than being perfectly horizontal, are apt at any given moment to be tilted through some small angle. The tilt will vary from moment to moment and from crystal to crystal.

In the everyday halo display, the tilts of the crystals are not especially small. In the simulation in Figure 1.4, for example, the tilts were a few degrees or so. (We are talking about the oriented plates and columns, of course—not the randomly oriented crystals.) That may sound small, but much smaller tilts are possible. As the tilts of the crystals become smaller, the common halos become brighter and more sharply defined, and rare halos, not visible in ordinary displays, begin to appear. In the best halo displays, the tilts of the crystals seem to be, almost incredibly, less than half a degree. These are the displays that the halo enthusiast dreams of, when the sky is filled with arcs of every description (Figures 1.8, 3.7, 6.7, 6.8).

Thus, one way to get uncommon halos is for the orientations of the oriented plates and columns to be uncommonly good, that is, for the crystals to be almost perfectly horizontal. Another way is for the crystals to take on uncommon falling modes—modes other than the common plate, column, and random modes (Chapter 6). A third way is for the halo-making crystals to occur in uncommon shapes—shapes other than hexagonal prisms. This turns out to be what was happening in Weickmann's display.

Ice Crystal Gallery

This chapter is a collection of ice crystal photographs. The collection is far from comprehensive, but it does illustrate two points. First, there are indeed real ice crystals that look just like the model crystals of Figure 1.5. Second, there are also many ice crystals that do not look like the model crystals. Most, in fact, do not.

All of the crystals in the photos were collected at ground level during cold weather, mostly in Alaska, Antarctica, Finland, or Siberia. A Petri dish, usually filled with hexane liquid, was exposed to the atmosphere for a few minutes, and crystals simply fell into the dish. They were then photographed through a microscope. The crystals shown in Figure 2.13 were handled differently: they were collected in liquid nitrogen and later photographed with an environmental scanning electron microscope (ESEM), with the nitrogen being allowed to boil off just before the photography.

The hexane is evident in many of the photos, where indentations in the basal faces of crystals exhibit heightened contrast due to air trapped inside. An example would be the lower left photo in Figure 2.7, where a large bubble is evident at the left in each of the two bullet clusters. The purpose of the hexane is to slow the deterioration of the crystals, but it has some drawbacks as well. It reduces contrast, making the photography more difficult, especially for thin plate crystals. It also tends to make crystals cluster together, as in the left-hand photo of Figure 2.3.

Good halos require that the crystals have fairly good planar faces and that the crystals not be too small. The crystals in Figure 2.2 look like great halo-makers, but the crystals in the right hand photo of Figure 2.3 do not. The tiny crystals in the background in Figure 10.13, most of which are less than ten microns in diameter, are too small to make halos; a swarm of such crystals gives only the appearance of fog.

People sometimes ask about the folk wisdom that no two snow crystals are alike. Probably if the crystals are to be subjected to unlimited scrutiny, then indeed no two will be exactly alike. But by the same standard, no two doorknobs, say, would be exactly alike either; small scratches or other signs of wear would distinguish between them. The point is that unlimited scrutiny is unreasonable. The crystal photographs here show that many crystals are in fact very much alike, especially the small simple prismatic crystals. Of course, when most people speak of snow crystals, they are not thinking about little prisms. Instead they have in mind something like the elaborate stellar crystals in Figure 2.1, and such crystals indeed seem to come in endless varieties; see, for example, the beautiful photographs in Kenneth Libbrecht's *The Snowflake* [42].

We mentioned that our photographs are not a comprehensive record of atmospheric ice crystals. Many more ice crystal photos can be found in the books and articles cited by Bailey and Hallett [2]. But bear in mind that nearly all ice crystal photos understate the number of spatial crystals and irregular crystals. This is partly because such crystals can be difficult to photograph, and partly because they tend not to catch the eye, so they get overlooked.

Photographs can only hint at the beauty to be seen in ice crystals, and photographs are no substitute for viewing the crystals yourself through a stereo microscope. While not everyone will have access to a microscope, large crystals can also be examined with a simple hand lens or reading glass, and they can be stunning.

FIGURE 2.1 Everybody's idea of an ice crystal?

FIGURE 2.2 Hexagonal and triangular plate crystals at left, solid hexagonal column crystals at right. These crystals resemble the model crystals of Figure 1.5. Having sampled atmospheric ice crystals during many halo displays, we know that crystals like these are good halo-makers. 1000 μ = 1 mm.

FIGURE 2.3 Good and bad halo-makers. The larger crystals at the left look like good halo-makers. The crystals at the right do not.

FIGURE 2.4 Mostly (hexagonal) plate crystals. Compare with the idealized plate crystal at the left in Figure 1.5.

FIGURE 2.5 (Hexagonal) column crystals. Compare with the idealized column crystal at the right in Figure 1.5.

FIGURE 2.6 More column crystals. More often than not, a column crystal will have an indentation in each basal face. The perspective in the photo at left, in which the crystal is leaning on another crystal, out of focus, nicely shows the hopper-like structure of the indentation. In all four photos the crystals are in hexane, but in the lower photos the indentations are mostly filled with trapped air, which heightens contrast and accentuates the indentations.

FIGURE 2.7 Bullet crystals—column crystals with one end tapered. Bullet crystals are common and can make 22° halos if the crystals are not too badly formed. Bullet crystals probably originate as clusters like those in the lower left photo. In the lower right photo one or two such clusters evidently broke apart upon impact with the collecting dish.

FIGURE 2.8 Stellar crystals. Their long dimensions—tip to tip—are 1150μ for the left-hand crystal and 1800μ for the right. Photos © Kenneth Libbrecht, snowcrystals.com.

FIGURE 2.9 Spatial crystals. (*Left*) Spatial plate. (*Right*) Column cluster with platelike appendages. One column, out of focus, points out of the page.

FIGURE 2.10 Pretty configurations of three columns.

FIGURE 2.11 Crossed plate crystals. Not a common crystal type.

FIGURE 2.12 Some uncommon crystal types. The two exotic crystal types illustrated in the upper left and lower right photos have been discussed by Bailey and Hallett [2]. In the lower left photo is a rare triangular column. The crystal in the upper right photo is a mystery to us.

FIGURE 2.13 Photographs of ice crystals taken with an environmental scanning electron microscope (ESEM). The black and white scale bars on the photos, clockwise from upper left, are 50, 100, 45, and 100μ. The crystals in the lower photos are not common types. Photos © Kenneth P. Severin.

1889. 7171.

Тагнѣла, oder ungewöhnliche Zeichen umb
Horizonte den 19 Aprilis Anno 1630 et
gesehen

die Sonnen, wie sie unter dem Münbergischen
Horizonte stand vor mittag, von nemmlich
gesehen worden

O Gott, o fromer Gott was zeigstū vns in gnaden!
 Wie treulich warneſt vns vor allgemeynen ſchaden!
 Will er dann helfen nicht ſo iſt das ſchmied gewelt,
 Und das tödlich geſchoß iſt vff die ſenn geſetzt.
 Zwar die natur hatt auch hierinn ihre vrsachen;
 Aber deß halb will ich kein ſpottwerck darauff machen.
 Ein ungewöhnlich ding iſt ſolch himmels figur;
 Gott droht durch wort v. werck. Gott droht durch die natur.

M.I.S.

4H 6192 177

Joh: pfann ſculp. et D.

The Beginnings of Halo Science

Presumably there were halos long before people. We do know that humans have long been aware of halos. In *The Rainbow: From Myth to Mathematics* [8], Carl Boyer writes of “cuneiform tablets of the Sumerian-Babylonian culture of four or five thousand years ago, in which names are given to the smaller halo radius of 22° (tarbasu) and the larger one of 45° radius (supuru).” Boyer mentions halos in connection with Aristotle (384–322 BC), Alexander (c. 200 AD?), Alhazen (c. 965–1039), and Theodoric of Freiberg (c. 1300), among others.

Some ancient paintings and even petroglyphs have been interpreted as depicting halo displays [27, 63]. The oldest unequivocal halo depictions that we know of, however, are not so old, with ages measured in centuries rather than millennia. The exquisite illustration in Figure 3.1 is from the early seventeenth century. It is by no means one of the oldest halo illustrations, but it is one that is easy to interpret, leaving no doubt that what is depicted is a halo display. The circle centered on the sun in the figure is of course the 22° halo, and outside it to the left and right are the parhelia. Just above the 22° halo is the tangent arc, and above the tangent arc is the rare Wegener arc. The large circle passing through the sun and through both parhelia is the parhelic circle. The artist had to abuse the perspective in order to show the entire sky and still include the city in the foreground, but otherwise the representation is excellent.

Although some of the common halos have been known since antiquity, satisfactory explanations of them were a long time in coming. René Descartes, for

FIGURE 3.1 Halo display in Nürnberg, April 19, 1630. German speakers will learn from the verse at the bottom that halos are not to be taken lightly. Staatsbibliothek zu Berlin – Preußischer Kulturbesitz YA 6192 kl, reproduced with permission.

instance, in his *Meteorology* [14] of 1637, attributed the parhelia to the presence of a giant ring of ice in the sky. Although his explanation sounds ludicrous to us today, Descartes himself apparently saw no problem with it; he concluded his *Meteorology* by saying that “I hope that those who have understood all that has been said in this treatise will, in future, see nothing in the clouds whose cause they cannot easily understand, nor anything which gives them any reason to marvel.” Yet Descartes was not stupid, and his explanation of the parhelia probably says more about the nearly complete ignorance of the atmosphere at that time, and about the absence of any competing explanation for the parhelia, than it does about his intelligence.

In any case, Descartes’ explanation of the 22° halo turned out to be more fruitful. He supposed that it formed in suitably shaped snow crystals high in the atmosphere; the crystals were thick in the middle and then tapered toward the edges. Such crystals could indeed make a 22° halo if they were shaped just right, but Descartes gave no details, and it is not clear to us exactly what he had in mind. The theory that he presented was purely qualitative, and we do not know whether he actually made the calculations that would have made his explanation more convincing. Nevertheless, to suggest that halos were due to ice particles was an important step. It prompted at least one other person to think along similar lines.

That person was Christiaan Huygens, and it seems to be Huygens who came up with the first quantitative explanation for halos. In his *Traité des Couronnes et des Parhélies* [33], written about 1662, he showed how the parhelia could arise in transparent water or ice cylinders having opaque cylindrical cores (Figure 3.2). Huygens assumed that the cylinders were floating in the air with their axes vertical and that the ratio m of the core diameter to the outer diameter was 0.48. Under these assumptions, you do indeed get something resembling parhelia, as shown in Figures 3.3 and 3.4.

Huygens also showed how these same cylinders, if floating with their axes horizontal instead of vertical, would give illumination from the regions above and below the 22° halo rather than from the regions to the right and left of it; such cylinders would thus explain the tangent arc. To account for the 22° halo, Huygens replaced the cylinders with spheres, each sphere being transparent and having an opaque spherical core, again with $m = 0.48$. For the circumzenith arc he used horizontal cylinders as for the tangent arc, but with m changed from 0.48 to 0.68. And for the 46° halo he used spheres, also with $m = 0.68$.¹

¹At the time that Huygens began his halo studies, the angular radii of the 22° and 46° halos were not known accurately. Huygens and others took them to be 22.5° and 45°, so that the angular diameters had the appealing values of 45° and 90°. Huygens originally used $m = 0.48$ and $m = 0.68$, from which he obtained the desired angular radii. When better measurements became available for the 22° halo, he replaced $m = 0.48$ with $m = 0.473$.

FIGURE 3.2 Huygens' parhelion maker, a transparent cylinder with an opaque cylindrical core. The sky was supposed to be full of such cylinders, all falling with their axes vertical, as here.

FIGURE 3.3 Celestial sphere with right and left parhelia (PH) arising in Huygens' cylinders. Here the sun and the parhelia are on the horizon. The tail of each parhelion is longer and brighter than in reality, but the crucial inner edge of each parhelion is in the correct position. The figure was made from a diagram like the middle one in Figure 3.4 but with more sun rays. Each dot corresponds to an outgoing ray from the cylinder.

FIGURE 3.4 Light rays forming the right parhelion according to Huygens. This is a top view looking straight down on three vertical cylinders, which therefore appear here as disks. The sun is on the horizon, and all light rays are in the plane of the paper. (*Left*) Ray paths through a transparent cylinder. The deviation Δ between the incoming and outgoing rays varies from 0° upward. (*Middle*) Same but with the addition of an opaque cylindrical core. Now Δ varies from 22° upward, with the densest concentration of rays at 22° . (*Right*) Same but showing only the least deviated ray together with a sun ray reaching the observer. The observer looks in a direction $\Delta = 22^\circ$ to the right of the sun to see the inner edge of the parhelion.

Edme Mariotte was the first to attribute halos to prismatic ice crystals. In his *De la nature des couleurs* [45] in 1681, he showed how the 22° halo could arise in randomly oriented columnar crystals, each crystal being in the shape of an equilateral triangular prism. Mariotte calculated light ray paths through crystals having various orientations and found a value of $22^\circ 50'$ for the minimum deviation between the incoming ray from the sun and the outgoing ray. A sky full of such crystals would show an abrupt increase in brightness—darker toward the sun, lighter away from it—at an angular distance of $22^\circ 50'$ from the sun; this was the 22° halo. To explain the parhelia, Mariotte used the same crystals but oriented them with their axes vertical.²

Mariotte's theory convinced hardly anyone. Did it even get a serious look from his contemporaries? We do not know, but if it did, it seems to have been soon largely forgotten. Smith's *Opticks* [69] of 1738, for instance, contains a twenty-nine page English translation of Huygens' work but never mentions Mariotte. Thomas Young, writing in 1807, claimed that by that time Mariotte's ideas on halos had been "almost entirely abandoned and forgotten."

Today Mariotte is not exactly a household name, but he was well known and highly regarded in his day. His theory of color, for example, is said [8] to have received more attention than that of his contemporary Isaac Newton. Today Mariotte is probably best remembered for his independent discovery of Boyle's Law, also known as Mariotte's Law. Mariotte's ideas on halos had been presented to the French Academy of Sciences in 1679 and had been published in 1681, then republished in Mariotte's collected works in 1717 and 1740. So they were available, whether or not anyone was paying attention.

Mariotte's ideas on halos were eventually revived—at the beginning of the nineteenth century, by Thomas Young [91] in England and Giambattista Venturi [83] in Italy. Since that time, Mariotte's basic idea—that halos form in polyhedral ice crystals in the atmosphere—has been universally accepted.

An imaginary debate

Today Huygens' theory, with its ad hoc cylindrical and spherical particles, seems preposterous. Yet it was apparently the prevailing theory of halos for more than a century. How could it have happened? We are not historians of science and we are not competent to say how it happened in reality, but we can at least muster the arguments for the two sides and try to imagine how a Huygens vs. Mariotte debate might logically have played out, had it ever come to pass. In doing so, we will be forced to think about how we know what we know.

²Mariotte's calculation of ray paths for the 22° halo applies more properly to the parhelia (with the sun on the horizon) than to the 22° halo, since the paths that he considered all lay in a plane perpendicular to the crystal axis.

Let us therefore consider the Huygens vs. Mariotte issue on its merits, but from the perspective of the eighteenth century. Just how preposterous were Huygens' ideas, and how preposterous were those of Mariotte?

For evidence for his layered spheres and cylinders, Huygens quoted Descartes' *Meteorology*: "...the outside of each grain of this sleet is usually composed of continuous and transparent ice, yet it has a bit of snow inside." Mariotte, on the other hand, pointed to the fern-like branches that occur on some snow crystals. He had apparently convinced himself that the tiniest extremities on these branches were triangular columns. Similar columns, perhaps unattached to any larger crystal, were supposed to be floating high in the atmosphere and causing the halos.

Today we know that Huygens' precisely layered spheres and cylinders do not exist, and therefore no one could have seen one. But we also know that triangular columns are so rare that nobody at the time would have seen one of them either. We do not find in the old literature any mention even of hexagonal columns, which are much more common than triangular columns and which are nearly equivalent in their optical effects. Robert Hooke had observed snow and frost crystals through the microscope and had published exquisite drawings of them in his popular *Micrographia* [28] in 1665, but there was nothing in his drawings to suggest the existence of prismatic columnar crystals, either triangular or hexagonal. Johannes Kepler [35] in 1611, Descartes [14] in 1637, Frederick Martens [46] in 1694, John Nettis [51] in 1756, and Johann Carl Wilke [90] in 1761 also described or drew snow crystals, but again there was no mention of hexagonal columns, though Descartes and Wilke had come close (Figure 3.5).

FIGURE 3.5 Ice crystal drawings by Descartes in 1637 at left, and Wilke in 1761 at right. Descartes' diagram F and Wilke's diagrams 17, 18, and 19 depict what are almost certainly capped column crystals—hexagonal columns with a hexagonal plate on each end. At the time there seems to have been no awareness of hexagonal columns, and the columnar parts of the capped columns here were understandably misinterpreted as cylinders.

FIGURE 3.6 Part of a page from the collected works of Huygens [33], showing the essentially correct theoretical dependence of the tangent arc on sun elevation Σ . Compare the upper left diagram with the much later diagram of Wegener in Figure 5.1 or with the real tangent arc in Figure 1.8. Here $\Sigma = 20^\circ, 0^\circ, 30^\circ, 10^\circ$, clockwise from upper left. The horizon is labeled in script.

Hexagonal columns are common in cold climates, where light snowfall sometimes consists predominantly of such columns. They can be big enough to see—sometimes a millimeter or so in length—but not big enough to see well, not without a decent microscope. In the few cases where they were seen prior to the nineteenth century, the columns were usually taken to be cylindrical rather than prismatic. The first unequivocal reports of hexagonal columns seem to be those of Scoresby, in his *Account of the Arctic Regions* [66], which appeared in 1820.

In the eighteenth century the direct observational evidence for Mariotte's particles was therefore about the same as that for Huygens'—virtually nil—and the choice between Mariotte's theory and Huygens' theory was not so clear. In fact, the edge might logically have gone to Huygens, as indeed it did, since he had accounted for many more halos than had Mariotte, and since he had worked out the implications of his theory in more detail. Huygens, in fact, was far ahead of his time. He knew, for example, the fundamental result of classical halo theory later known as Bravais' law, and he had used it to calculate the appearance of the parhelia and the tangent arc as a function of sun elevation, all of this nearly two centuries before Bravais (Figure 3.6).

FIGURE 3.7 Beautiful lunar halo display, Jämsä, Finland, November 22, 2004. Compare the lower tangent arc here with Huygens' diagram at the lower right in Figure 3.6. Photo © Arto Oksanen.

Mariotte himself had treated only the 22° halo and the parhelia, but near the beginning of the nineteenth century his ideas were extended to explain other common halos. Young [91] in 1807 explained the tangent arc, and Young together with Cavendish explained the 46° halo. In 1840 G. Galle [18] explained the circumzenith arc.³ Each of these explanations attributed the halos to hexagonal prismatic crystals having suitable orientations.

By this time both Huygens' theory and Mariotte's theory therefore offered explanations for most of the common halos: the 22° and 46° halos, the parhelia, the tangent arc, and the circumzenith arc. For these halos the predictions of the two theories are not much different. The predicted intensity distributions differ from one theory to the other, but the critical location of the inner, i.e., sunward,

³Galle's explanation was not correct. Galle, and indeed Huygens, confused the circumzenith arc with the upper tangent arc of the 46° halo, a largely hypothetical arc sometimes known as Galle's halo. (In the terminology of later chapters this halo is one of the 46° Lowitz arcs.) We have included Galle's explanation, somewhat artificially, in order to emphasize the similarities between Mariotte and Huygens. By 1840, when Galle's work appeared, Huygens' theory had already fallen from favor, even without having to face a tenable competing explanation of the circumzenith arc from Mariotte's side.

edge of each halo, where the halo tends to be brightest, turns out to be exactly the same for the two theories.

So in the early nineteenth century the choice between Huygens and Mariotte must have hinged mainly on the plausibility of the two ice particle models—hexagonal prisms versus layered spheres and cylinders—and there was still little direct observational evidence for either model. In the eighteenth century, when Mariotte's theory was incompletely worked out and therefore offered less competition, the difficulty of conceiving Huygens' spheres and cylinders with their ad hoc m -values probably would have been overlooked. But as Mariotte's theory was fleshed out, Huygens' spheres and cylinders must have seemed increasingly contrived, with the concocted m -values of 0.48 and 0.68 demanding explanation. By contrast, in Mariotte's theory the role of these two parameters is played by the natural interfacial angles of 60° and 90° on the hexagonal prisms, and no apologies are needed.

When one attempts to extend the theories of Huygens and Mariotte to the less common halos—most of which were little known and therefore not relevant at the time—Huygens' particle model fails badly and Mariotte's succeeds impressively, as we will see.

But you needn't take our word for it regarding the cause of halos, at least not if you live in a cold enough place. The north central United States will do, and so will northern Scandinavia. Just wait for one of those clear cold winter mornings when the air is filled with ice crystals. The crystals will be obvious, sparkling in the sunlight as they fall, but the sparkles may be more evident in some directions than in others. Those directions are the directions of halos forming in low level crystals, more or less in front of your nose. The halos can sometimes be bright enough to appear in front of nearby buildings or other objects (Figure 3.8). It will be obvious that halos have something to do with ice crystals and sunlight.

But what sorts of crystals are they? Again you can see for yourself, but you will need a microscope or strong hand lens unless the crystals are unusually large. Just expose a cold glass slide or dish for a few minutes to gather a sample of the falling crystals, and then take a look. If you were seeing good halos in your sparkles, you will normally find some hexagonal prisms—plates or columns—in your sample. For those of us living in cold places it is easy to believe that halos arise in prismatic ice crystals. Mariotte was right.

Although Mariotte was right about the responsible ice particles, it was Huygens who better understood the role of the orientation of the particles, and it was Huygens who had worked out the mathematics of the refraction of light rays in three dimensions. The theory of halos that developed at the beginning of the nineteenth century, and which forms the core of the modern theory, was a melding of the ideas of Mariotte and Huygens.

The failed ice particle model of Huygens is a nice reminder that much of science is an exercise in the logical fallacy known as affirming the consequent. (From the statements $p \rightarrow q$ and q , you infer p .) We are not complaining or criticizing; this is just the way science works. As an illustration, let's try to put ourselves again in eighteenth century shoes: We probably have not had a good look at an ice particle through a microscope, and we probably have never heard of Mariotte's theory. Who among us could look at Huygens' very explicit predictions for the tangent arc as a function of sun elevation (Figure 3.6), then see those same distinctive and unintuitive shapes confirmed in real tangent arcs (e.g., Figures 1.8 or 3.7), and then doubt Huygens' particle model? And of course it was not just the tangent arc; Huygens had made correct predictions for the other common halos as well. You may protest that you see no way for Huygens' particles to arise. Huygens did try to explain the origin of his particles (not very convincingly), but we suspect that he might also have responded much as did Alfred Wegener, who, when criticized for having no mechanism to explain his proposed continental drift, replied to the effect that, never mind, a mechanism will be found [82]. The point is that when the predictions resulting from a model conform so well to reality, that is, when "the consequent is affirmed," you tend to believe the model. As Huygens put it,

...at last I hit upon their [the halos'] true cause... For I make no scruple to call those causes true, whose effects agree so exactly with observations, as to make it seem unnecessary to search for others. (Smith's *Opticks* [69, page 200])

But there is always a tiny chance that the model is wrong, even if a great many of its consequences have been confirmed. And that was the case with Huygens' ice particle model.

Anyone interested in learning more about the early theory of halos will enjoy reading Auguste Bravais' 1847 *Mémoire sur les Halos...* [9]. For many years the *Mémoire* was quite literally the last word on halo theory, with virtually no important halo publications appearing again until the beginning of the twentieth century. Today, a century and a half after its publication, the *Mémoire* remains a classic reference.

FIGURE 3.8 (*Opposite*) Evidence that halos are caused by ice crystals. In this low level halo display, discrete sparkles from nearby crystals are seen in the directions of the upper and lower tangent arcs. The lower tangent arc is showing up only as a concentration of sparkles between the photographer and the hut. South Pole, February 16, 1986.

FIGURE 3.9 More evidence that halos are caused by ice crystals. Here the 22° halo is forming in ice crystals on the grass. Eau Claire, Wisconsin.

How Halos Form

Halos form when sunlight falls on suitable ice crystals in the atmosphere. But how exactly does it happen?

The sun rays are parallel

In general, it can be difficult to see why a given halo looks the way it does, and we will be grappling with this problem as we go along. The problem is simplified if we assume that all light rays from the sun are parallel. The sun therefore appears as a point in the sky and does not change its direction as an observer changes position. These assumptions are not far off; the angular diameter of the sun is indeed small, and the sun is far away. Actually the first assumption—that the sun appears as a point—is not crucial, and we can easily cope with a sun having positive angular diameter. We nevertheless assume, for the moment, that all sun rays are parallel. The sky is filled with them.

Light rays and light points

When thinking about halos, we often care as much about the light point of a light ray as we do about the ray itself. The *light point* of the ray is the point on the celestial sphere in the direction opposite to the ray. It is the point that appears to be lit, should the ray reach the eye of an observer.

The simplest halo display

What if the sky could somehow be filled with crystals all having exactly the same shape and same orientation? Suppose, for example, the crystals were all shaped

FIGURE 4.1 A common ray path through an oriented plate crystal. The ray from the sun enters the top basal face of the crystal and exits a prism face. This ray path contributes to the circumzenith arc. The arrowheads are drawn as solid cones so as to convey the directions of the rays in space; compare with Figure 4.2.

and oriented as in Figure 4.1. What would an observer see in the sky?

To answer such questions, you always need to think about the way light rays from the sun pass through the crystals. Consider, for example, a ray path as in Figure 4.1. The ray from the sun enters the upper basal face of the crystal and exits a prism face. The ray is refracted downward as it enters the crystal and is refracted once more as it exits.

The chance that the outgoing ray from this particular crystal reaches the eye of our fixed observer is nil. But throughout the sky the sun rays are parallel and the crystals are identical, so somewhere there will be a crystal having a ray path whose outgoing ray does reach the observer, and whose ray path is parallel, segment for segment, with the original ray path (Figure 4.2, upper right). The observer perceives a point of light in the direction opposite to that of the outgoing ray; this point is the light point of the outgoing ray, also known as the *halo point*. The halo display due to rays like those in Figure 4.1, passing through these identically shaped and oriented crystals, would consist of this single spot of light in the sky (Figure 4.2, bottom left).

Thus, in thinking about how halos form, you need only consider a single crystal, not a whole sky full of crystals. So long as every small region of sky is equivalent to every other, you need not worry about where the observer is nor whether the outgoing ray from the crystal actually reaches the observer. This is a big simplification.

A more realistic halo display

Let's now take the same crystal and rotate it about its axis, with the axis remaining vertical. As the crystal rotates slightly, the path of the outgoing ray changes slightly, and the halo point in the sky changes slightly as well. As the crystal continues to rotate, the halo point traces out a halo.

This halo happens to be the circumzenith arc. The responsible crystals are oriented plates, that is, plate crystals with their axes vertical. The responsible light

FIGURE 4.2 (Top left) Same crystal and ray path as in Figure 4.1 but seen from a slightly different viewpoint, one for which the halos can be more easily depicted on the celestial sphere. (Top right) Sky filled with crystals having shapes and orientations the same as at top left. An outgoing ray from one of the crystals reaches the observer. (Bottom left) Sun point **S** and halo point **H** on the celestial sphere. The halo point is the light point of the outgoing ray, that is, the point of the celestial sphere that appears lit to the observer. The crystal is at the center of the sphere. (Bottom right) Same as bottom left except that the crystal rotates about its (vertical) axis. As the crystal rotates, the halo point traces out a halo, the circumzenith arc **CZ**.

FIGURE 4.3 Another ray path through the oriented plate crystal of Figure 4.1. The ray enters a prism face and exits an alternate prism face. This ray path contributes to the left parheliion. (*Left*) The same perspective as in Figure 4.1. (*Right*) Top view of the same crystal and ray path.

rays are those that enter a top basal face and exit a prism face, as in Figure 4.1. It is clear from the figure, in which the ray is deviated sharply downward, why the circumzenith arc appears nearly overhead; this is the direction that an observer must look in order to see where the outgoing ray appears to be coming from.

The first halo display that we considered, consisting of a single spot of light, is of course entirely imaginary; you will never get the necessary sky full of identically shaped and oriented crystals. But the second display, arising in the rotating crystals, is another matter. Although you cannot expect a sky full of synchronized rotating crystals, you can expect something equivalent, namely, a sky full of oriented plate crystals. That is, each crystal should have its axis (more or less) vertical, and all rotational orientations about the axis should be equally likely. Individual crystals need not be rotating.

Different ray paths make different halos

Let's return now to the first halo display, the imaginary display produced by the identically shaped and oriented crystals. The ray path that we considered (Figure 4.1) is not the only way that a sun ray might pass through the crystal. A ray might, for example, enter a prism face and exit another prism face as in Figure 4.3. The new outgoing ray would have an entirely different direction from that of the outgoing ray for the circumzenith arc, and hence it would have an entirely different halo point on the celestial sphere. We see that the halo display would actually consist of at least two points of light, not just one.

If, as before, we let the crystal rotate about its axis, then the new halo point traces out a new halo, which happens to be the left parheliion (left, as seen from inside the celestial sphere). The two different ray paths cause two different halos.

Still other ray paths through the same oriented plate crystal are possible. Perhaps a dozen of them have outgoing rays that are bright enough to be significant. The first display that we considered, with a fixed crystal orientation,

actually consists of about a dozen isolated spots of light in the sky; these are halo points of various outgoing rays. In the second display, where the crystal is allowed to rotate about its axis, each halo point traces out a halo, thus giving about a dozen halos. In general, a halo is the locus of the halo point of a specified ray path as a crystal takes on orientations in the given orientation class. Change the ray path and you (usually) get a new halo.

Both displays are shown in Figure 4.4. In making the right hand diagram, we only allowed the crystal to rotate through an angle of 15° (in increments of one degree), so as to keep the diagram from becoming impossibly busy. Most of the halos are therefore incomplete. The parhelic circle, for instance, if complete, would be a circle going all the way around the sky at the elevation of the sun.

In the figure each ray path is written as a sequence of numbers, namely, the numbers of the crystal faces that the ray encounters. The basal faces of the crystal are numbered 1 and 2, and the prism faces are numbered 3, 4, 5, 6, 7, 8, in counterclockwise sequence when looking at face 1. The face numbers are shown in Figure 4.4 and more clearly in Figure 6.1. But the details of the ray paths are not so important at the moment. The main point here is that different ray paths give different halos.

FIGURE 4.4 Same as the bottom diagrams of Figure 4.2 but with halo points from additional ray paths. (*Left*) The crystal orientation is fixed, the same as in the bottom left diagram of Figure 4.2. The resulting imaginary halo display consists of about a dozen isolated halo points. Each halo point is labeled with its ray path. (*Right*) The crystal is allowed to rotate. Each halo point in the left diagram begins to trace out a halo. **CZ** = circumzenith arc, **PH** = parhelia, **PHC** = parhelic circle, **120 PH** = 120° parhelia, **SUBPH** = subparhelia, **SUBPHC** = subparhelic circle.

There are exceptions. According to Figure 4.4, the two ray paths 1 (reflect off the top basal face) and 326 (enter a prism face, reflect off the bottom basal face, and exit the prism face opposite the entry face) both contribute to the halo known as the subsun. This is clear if you can see that the direction of the outgoing ray is the same for both ray paths. Another example: The ray paths 35 and 46 both contribute to the left parhelia. But here the distinction between the two paths is superficial, being essentially an accident of the face numbering.

The point remains that different ray paths usually do make different halos. The existence of many essentially different ray paths is one factor that makes for a rich variety of halos.

Why the halos look the way they do

This chapter was supposed to tell how halos form. Perhaps it did, but it did not explain why the halos look the way they do. To do so, we need to be able to find the paths of the light rays through the crystals, so that we can in turn find the halo points on the celestial sphere. What is needed are the laws of reflection and refraction, but in a geometrical, conceptual form.

In order to express these laws simply, we revise slightly our notion of light point. The light point of a ray will still be the point on the celestial sphere that the ray appears to light, but the celestial sphere will now be regarded as having radius equal to the refractive index of the medium containing the ray. Thus there must be a separate celestial sphere for each refractive index. All of them are concentric.

Laws of reflection and refraction *Suppose a light ray is incident on a boundary between two media. Then the light points $\mathbf{S}$, $\mathbf{S}'$, and $\mathbf{R}$ of the incident, reflected, and refracted rays all lie on a line normal to the boundary (Figure 4.5).*

Thus to get $\mathbf{S}'$ and $\mathbf{R}$ from $\mathbf{S}$, you just project $\mathbf{S}$ in a direction normal to the boundary; the point $\mathbf{S}'$ is on the same sphere as $\mathbf{S}$ but on the opposite side of the boundary, and the point $\mathbf{R}$ is on the other sphere but on the same side of the boundary.

Consider as an example the ray path 1 in oriented plate crystals; it is just a reflection off the top basal face of the crystal (Figure 4.6). What is the resulting halo? The incoming and outgoing rays are both external to the crystal, and so their light points—the sun point $\mathbf{S}$ and the halo point $\mathbf{H}$ —are both on the sphere of radius 1 (refractive index of air). The boundary between the two media (air and ice) is the top face of the crystal, which is horizontal. The normal direction is therefore vertical, and so $\mathbf{H}$ is directly below $\mathbf{S}$, at the so-called subhelic point. The halo is the subsun.

FIGURE 4.5 How to find refracted and reflected rays. (Left) Incident, reflected, and refracted rays S , S' , and R at a boundary between media having refractive indices n_1 and n_2 . The boundary is perpendicular to the plane of the paper, and all rays are in the plane of the paper. The dashed ray R' is just to complete the symmetry; it is the ray that would reflect to R and refract to S' . (Right) The corresponding light points S , S' , R and R' . The light points all lie on a line normal to the boundary, and any one of them therefore determines the others.

FIGURE 4.6 Formation of the subsun by a ray reflecting off the top face of an oriented plate crystal. According to the law of reflection, the halo point H must be on the same sphere as the sun point S and directly below it, since the normal to the reflecting face is vertical. The right-hand diagram is the "light point diagram" for the subsun. The crystal is at the center of the sphere.

The subsun is a common halo, perhaps the most common halo, but how many people have seen it? The problem in seeing it, is that you need to be above the ice crystals. Either you need to be in a place cold enough that atmospheric ice crystals can occur near the ground, or you need to fly. But whenever you can manage to have sunlit crystals below you, look for the subsun. It is just a bright spot below the sun, exactly as far below the horizon as the sun is above it.

For a second illustration of the laws of reflection and refraction, refer to Figure 4.7, which shows how the circumzenith arc is formed. Now two spheres are

FIGURE 4.7 Light point diagram for the circumzenith arc, showing the formation of the arc. (*Left*) Light points **S**, **R**, and **H** of the entry, internal, and exit rays for the ray path of Figure 4.1. The halo point **H** is found by projecting the sun point **S** to the outer sphere, thus getting **R**, and then projecting **R** to the inner sphere, getting **H**. See text. (*Right*) Same but for many different rotational orientations of the crystal about the vertical. The halo point traces out the circumzenith arc **CZ**. The spheres have radii 1 and 1.31, the refractive indices of air and ice.

required—an inner sphere of radius 1, to accommodate light points of external rays, and an outer sphere of radius 1.31 (refractive index of ice) to accommodate light points of internal rays. For a given crystal orientation the halo point is found by first projecting the sun point to the outer sphere, then projecting the resulting point back to the inner sphere. Each projection direction is normal to the relevant crystal face; the first projection is vertical, since the entry face is horizontal, and the second projection is horizontal, since the exit face is vertical (Figure 4.1). From Figure 4.7 it is easy to see why the circumzenith arc is part of a horizontal circle; the arc must lie on the intersection of a horizontal plane with the celestial sphere.

Figures 4.6 and 4.7 are “light point diagrams.” They show how to construct the halos, starting with the crystal shapes and orientations and using the laws of reflection and refraction. If you understood those figures, you may enjoy thinking about the light point diagrams for other ray paths or other crystal orientations. Try, for example, the ray path 326 in oriented plate crystals; it contributes to the subsun. Or try the ray paths 1 or 326 in oriented column crystals; they contribute to the parhelic circle.

Light point diagrams for a few more halos appear in Chapter 6. For many other halos, however, we will have to rely on simulations or on “pole theory” to understand them, rather than on their light point diagrams. It is not that their light point diagrams fail to be correct. Rather, they become too complex and not very enlightening. The diagrams can be drawn by computer, and they are accurate, but they can still be hard to make sense of, sometimes just a jumble

FIGURE 4.8 Derivation of the geometric version of the law of refraction. (*Left*) Incident and refracted rays as in Figure 4.5, drawn with lengths of n_1 and n_2 , respectively. The traditional law of refraction, namely, $n_1 \sin \theta_1 = n_2 \sin \theta_2$, ensures that the tangential components of the two rays are equal. (*Right*) Same but showing the light points **S** and **R** of the incident and refracted rays. The line segment **SR** is normal to the boundary, as claimed. The celestial spheres, not shown here, are centered at the point **O**.

of lines. Even a halo as common and relatively simple as the tangent arc turns out to have a fairly complex light point diagram. So after Chapter 8 you will not see any more light point diagrams.¹ But the fundamental laws of reflection and refraction will always be there, incorporated somewhere in the theory.

The geometrical versions of the laws of reflection and refraction that we have been using are not as widely known as they deserve to be, though they date back at least to Hamilton [23] and MacCullagh [44] in the nineteenth century. The geometric version of the law of refraction is derived in Figure 4.8.

¹Not all the halos of later chapters are complicated. The upper 23° plate arc (Chapter 15) would be an example, its light point diagram being similar to that of Figure 6.9.

FIGURE 5.1 Tangent arc as calculated by Wegener [85] in the 1920's. The responsible crystals are oriented columns, and the ray paths are those that enter and exit alternate prism faces. The sun elevation is 20° . Wegener used stereographic projection from the zenith, which gives the halo a slightly unfamiliar look. The significance of the two red segments is explained on page 52.

FIGURE 5.2 Computer simulations showing halos from oriented columns. The sun elevation and the projection are the same as in Figure 5.1. (Left) Allowing only ray paths for the tangent arc, that is, ray paths that enter and exit alternate prism faces. The tilts of the crystals are zero. (Middle) Allowing all ray paths. The tilts of the crystals are still zero. (Right) Allowing all ray paths. The tilts of the crystals are several degrees.

Halo Simulations

We understand halos far better today than thirty years ago. Several factors are responsible, but none more significant than the simulation of halos by computer. Prior to the computer, although it was quite clear in principle how to predict the halos due to specified crystal shapes and orientations, in practice the calculations were often too cumbersome to be feasible. With the computer the calculations became effortless. The results, moreover, could be displayed graphically in simulations that *looked* like real halo displays.

A comparison of Figures 5.1 and 5.2 gives some suggestion of the power that halo simulations can bring to bear. Figure 5.1 is an example of a theoretical halo prediction made prior to the computer. It is a relatively sophisticated example, taken from Alfred Wegener's elegant *Theorie der Haupthalos* [85] from about 1925. The halo in the figure is the tangent arc, calculated for a sun elevation of 20° . Wegener was able to calculate the entire region of sky occupied by the halo, not just the so-called caustic curve, where the halo tends to be brightest. This depiction was about as much as anybody could hope for at the time.

Figure 5.2 shows three computer simulations analogous to Wegener's diagram. The simulation in the left-hand diagram is the closest to Wegener's diagram; in both diagrams the responsible crystals were oriented columns, the tilts of the crystals were zero, and the allowed ray paths were only those that make the tangent arc. The two diagrams agree with regard to the region of sky occupied by the halos, but the computer simulation enjoys a crucial advantage in that, unlike the Wegener diagram, it indicates intensity variations within the halo. In general, the simulation gives a more realistic and appealing depiction of the halo.

The middle simulation in Figure 5.2 allows all ray paths through oriented columns, not just the ray paths that make the tangent arc. It therefore shows all

FIGURE 5.3 Point (heavy dot) on the celestial sphere in the direction of the crystal axis. The point gives a partial depiction of the crystal orientation. The crystal is at the center of the sphere.

FIGURE 5.4 Depiction of the crystal orientations for the simulation in Figure 1.4. Each of the points shown on the sphere is determined by a crystal orientation as explained in Figure 5.3. (*Left*) For the plate orientations. (*Middle*) For the column orientations. (*Right*) For the random orientations.

FIGURE 5.5 Depiction of the crystal orientations for the simulation in Figure 1.7.

halos (in the given region of sky) due to the crystals under consideration. This was not possible prior to the computer, since there was no way to anticipate all of the ray paths that would be significant. And even for the paths that could be anticipated, it was difficult to judge their relative importance in making the halos. This diagram is a big step forward.

The right-hand simulation is the same except that, instead of being perfectly horizontal, the crystals were given tilts of several degrees. The simulation shows immediately the effect of the larger tilts. While some of this information could also have been obtained from hand calculations, the computer simulation is far more effective in displaying the results.

Seeing crystal orientations

The computer is also good at displaying the crystal orientations that go into a simulation. The resulting picture is usually more enlightening than an analytic description of the orientations. Figure 5.3 shows how a crystal orientation can be partially represented as a point on the celestial sphere. In this way any set of crystal orientations appears as a set of points on the sphere.

Figure 5.4 is the picture for the crystal orientations that went into making the simulation in Figure 1.4. The column orientations appear as a band of dots around the equator, and the plate orientations appear as a concentration of dots centered at the north pole. The random orientations are of course spread uniformly over the sphere.

Figure 5.5 is the same, but for the much better halo display in Figure 1.7. The crystals obviously have much smaller tilts; the dots for the column orientations are barely distinguishable from the equator, and the dots for the plate orientations are clustered tightly about the north pole.

Revealing the ray paths

Figure 5.6 is an all-sky simulation of halos due to oriented columns. The simulation contains 20,000 halo points, each of which arises from a ray path through some crystal. The computer can remember the ray path for each halo point and can be instructed to sort and count ray paths, thus showing which types of ray paths were responsible for the halos. The table in the figure shows the result of sorting the 20,000 ray paths. For example, the tenth line of the table shows that 2.8% of the 20,000 ray paths fell within the broad category of 1352 (Again see Figure 6.1 for face numbering.)

But which ray paths made which halos? We next show by example how the computer can help to answer this question. As we do so, please bear in mind that,

FIGURE 5.6 (Left) All-sky simulation of halos made by oriented columns. There are 20,000 halo points in the simulation. (Right) A sorting of the 20,000 ray paths. Path categories with less than one percent of the total are omitted from the table. The sorting used here is rather coarse, in the sense that the criterion for grouping of ray paths is relatively easy to satisfy; this keeps the table small but hides some information.

FIGURE 5.7 The halo points in Figure 5.6 whose ray paths are in the categories 1352 (left) and 1372 (right).

although we give some details in order to make the example more concrete, the details themselves are unimportant at the moment.

Continuing, then, with the ray paths in the category 1352 mentioned above, we can ask the computer to plot just the halo points that these paths made. The halo points turn out to lie on the left side of the halo known as the subhelic arc. Similarly, the halo points with paths in category 1372 lie on the right side of the subhelic arc. See Figure 5.7.

Thus the ray paths in categories 1352 and 1372 made the subhelic arc. But were there other paths that also contributed? If we were to forget the computer for a moment and just apply the laws of reflection and refraction (page 38) to the paths 1352 and 1372, we would discover a characteristic property of the halo made by these paths: the halo point can always be found by rotating the sun point through an angle of $\pm 120^\circ$ about the crystal axis, with the crystal axis of course being thought of as passing through the center of the celestial sphere. Neither the property itself nor the details of its justification are at all important at the moment; what is important is that there is some criterion—independent of ray path—for deciding which points of the celestial sphere are on the subhelic arc. The computer can therefore be told to sort just those ray paths whose halo points are on the subhelic arc.

The results of such a sorting are shown in the table in Figure 5.8. There were 1,486 halo points on the subhelic arc. The bulk of their ray paths, about 76%,

FIGURE 5.8 (Left) Same as the simulation in Figure 5.6 but only plotting halo points on the subhelic arc locus; the points are chosen according to location, not according to ray path. There are 1486 such points. (Right) Sorting of the 1486 ray paths. Path categories with less than one percent of the total are omitted.

do indeed fall within the two categories 1352 and 1372. There are also some paths—in the first five lines of the table—that appear in the table by accident, in the sense that their halos happen to intersect the subhelic arc; these paths are irrelevant to the subhelic arc. But there are also two lines of the table—the last two—that reveal new ray path categories for the subhelic arc, namely, 134562 and 165432. That these two ray paths are genuine subhelic arc ray paths can be seen either by plotting their halo points—from among the original 20,000 halo points—or by realizing that the two paths always satisfy the geometrical criterion for their halo points to be on the subhelic arc.

The sorting that was used in making the tables in Figures 5.6 and 5.8 was intentionally made to be rather coarse. For example, the sorting considers the two ray paths 1352 and 1382 to be the same and therefore puts them both in the same category, namely, 1352. There is some justification for doing so, since the two internal reflections 3 and 5 have the same effect on the light point as 3 and 8. However, in the first case the two internal reflections are from alternate prism faces, while in the second they are from adjacent prism faces, so that for some purposes you might want to distinguish between the two paths. Moreover, if the computer were asked to take the 20,000 halo points from before and to plot first those that have ray paths like 1352, and then those that have ray paths like 1382, you would find that the two ray paths tend to light different parts of the subhelic arc, so that again there is some reason to distinguish the two paths. When we want to understand a halo in detail, we therefore use a finer sorting than that in the two tables above.

The subhelic arc is an interesting halo. Wegener [85] was aware that the existence of the subhelic arc was a theoretical possibility, and he was able to calculate the expected shape of it using the geometrical characterization mentioned above, but he thought the arc would be too weak to be visible. He was very nearly right; the subhelic arc is a very rare halo. Its existence, however, is no longer in doubt. It is common in the big halo displays in the Antarctic interior (Figures 6.7 and 6.8), and it is seen on rare occasions elsewhere as well (Figure 5.9).

How the simulations are made

The first computer simulations of halos were made by Robert Greenler [21] and his colleagues in the 1970's. Modern simulations are closer to those of Pattloch and Tränkle [56] from the 1980's.

In making a halo simulation the crystal shape is specified in advance. The crystal orientations are specified as well, perhaps with some orientations being more likely than others. The problem, then, is to display the halos that are theoretically expected from the specified crystal shape and orientations.

FIGURE 5.9 Halo display with remarkably strong subhelic arc. Fairbanks, Alaska, March 6, 2002. The responsible crystals were at normal cloud levels, not near the ground.

The solution is much in the spirit of Chapter 4. A sun ray is allowed to fall on a suitably oriented crystal, the ray is followed through the crystal, and a dot is plotted at the halo point. The process is then repeated. After many repetitions, halos begin to appear as concentrations of dots on the celestial sphere.

Of course, all of the preceding is going on mathematically. The crystal is described mathematically, the light rays are described mathematically, and so forth. There are no real crystals, no real light rays.

There are some differences between what the computer does in making the simulations and what was described in Chapter 4. In making the simulations, no ray path is specified in advance. When a ray from the sun encounters a

face of the crystal, it gives rise to a reflected ray outside the crystal and a transmitted ray within the crystal. The computer decides which of the two rays to follow by calculating their intensities and treating the (normalized) intensities as probabilities. If, for example, the reflected and transmitted rays have intensities of .1 and .9, respectively, then ten percent of the time the computer will choose the reflected ray to follow, and ninety percent of the time it will choose the transmitted ray. If the reflected ray is chosen, then its light point is plotted in the simulation. If the transmitted ray is chosen, then it is followed within the crystal until it encounters a second crystal face. Again there will be a transmitted and a reflected ray, and again the computer decides which to follow according to their probabilities. If the transmitted ray (now external) is chosen, then its light point is plotted in the simulation. If the reflected ray (now internal) is chosen, then it is followed to the next crystal face, and so on. As soon as a halo point is plotted, the computer stops and goes on to consider another sun ray and another orientation for the crystal, and the process repeats. When large numbers of dots are plotted, the density of dots in a given region gives an indication of intensity. The simulations show intensity variations within a halo as well as intensity variations from one halo to another.

The computer program that makes the simulations is rather powerful. For example, it can be adapted to simulate halos made by crystals of any polyhedral shape, convex or not, even if the crystals are strongly birefringent. It can therefore simulate halos made by crystals of virtually any mineral, not just ice. The program is not magic, of course; the programmer must still tell the computer what mineral, what crystal shapes, and what crystal orientations to work with.

Halos From Prismatic Crystals

This chapter treats what might be called the classical halos—the halos arising in the simplest sorts of ice crystals, namely, hexagonal prismatic crystals. These halos, then, are not the odd radius halos. The treatment is not intended to be comprehensive. More information, including ray path diagrams and many more simulations, can be found in the book *Atmospheric Halos* [76]. Also, at the time of this writing there was at least one halo simulation program available on the internet.¹ With it, you can change the sun elevation, the crystal shapes, crystal orientations, etc., and see for yourself how the halos are expected to change.

The halos in this chapter are organized according to the crystal orientations that make them (Figure 6.1). By far the most common orientations are plate orientations, column orientations, and random orientations. Parry orientations are rare, and Lowitz orientations are rarer still.

Plate arcs

Recall that a crystal is in plate orientation when its two basal faces are (more or less) horizontal. Numbering the crystal faces 1, 2, . . . , 8 as in Figure 6.1, we always assume—without loss of generality—that in plate orientation it is face 1 that is the top horizontal face. The halos that arise in crystals with plate orientations are known as *plate arcs*. Plate arcs from prismatic crystals are shown in Figure 6.2. The 22° plate arcs are the plate arcs that have the same ray path as the 22° circular halo, and the 46° plate arcs are those that have the same ray path as the 46° circular halo.

¹<http://www.sundog.clara.co.uk/halo/halosim.htm>. This website has much more on halos, including many fine photographs.

FIGURE 6.1 (Far left) Hexagonal prismatic crystal with standard face numbering. The basal faces are numbered 1 and 2, and the prism faces are 3, 4, 5, 6, 7, 8, in counterclockwise sequence when looking directly at face 1. (Center left) Same crystal but with plate orientation; face 1 is horizontal and on top. (Center) Same but with Parry orientation; face 3 is horizontal and on top. (Center right) Same but with column orientation; faces 1 and 2 are vertical. (Far right) Same but with Lowitz orientation. The indicated axis remains horizontal.

Plate arcs can be understood using the laws of reflection and refraction (page 38), as was done for the subsun and circumzenith arc in Figures 4.6 and 4.7. The parhelic circle and the circumhorizon arc are easy, the parhelia and the 120° parhelia require more thought.

You will find that plate arcs from prismatic crystals are confined to at most four horizontal circles on the celestial sphere. For the sun elevation of Figure 6.3 the circles C_1 and C_2 —the parhelic and subparhelic circles—would contain the parhelia and subparhelia, respectively, and the circles C_3 and C_4 would contain the circumzenith arc and the (not yet observed) subcircumzenith arc. Plate arcs lie on these four circles because all of the projection directions called for by the laws of reflection and refraction are either horizontal or vertical.

Column arcs

Recall that a crystal is in column orientation when the crystal axis is horizontal. The halos that arise in crystals with column orientations are known as *column arcs*. Column arcs from prismatic crystals are shown in Figure 6.4.

In principle you can understand most column arcs if you understand plate arcs. We now describe briefly how this goes, but mainly for historical reasons; it is not something that we need later in the book. The idea is that if you fix the crystal axis and let the crystal spin about the axis, you get “rotated plate arcs.” These arcs are like plate arcs but with the point on the celestial sphere in the direction of the crystal axis playing the role of the zenith. (Forget about the horizon, pretend that “up” is in the direction of the crystal axis.) This is the idea behind Wegener’s depiction of the tangent arc (our Figure 5.1). There the two red segments are the “rotated parhelia” made by the ray paths 35 and 37 as the crystal spins about its axis. These rotated parhelia lie on a circle through the sun (not shown), but the circle is vertical, not horizontal, since its center is a point on the horizon—the point

Halo		Ray path	
22° plate arcs	Left and right parhelia	PH	35, 37
46° plate arcs	{ Circumzenith arc Circumhorizon arc	CZ	13
		CH	32
Arcs involving reflection	{ Parhelic circle Left and right 120° parhelia	PHC	3, 132
		120 PH	1342, 1382

FIGURE 6.2 All-sky simulations showing plate arcs from prismatic crystals. The crystals are shaped like the one shown. Sun elevations $\Sigma = 20^\circ, 60^\circ$. The table gives common ray paths for the halos.

FIGURE 6.3 The four horizontal circles C_1, C_2, C_3, C_4 (heavy segments) of the celestial sphere where plate arcs from prismatic crystals can occur. Here the spheres of radii 1 and $n = 1.31$ are seen in cross section, with the sun at **S**. The circles change with sun elevation, but there are always at most four.

in the direction of the crystal axis. (If you rotate the celestial sphere, including the sun, in such a way that the circle becomes horizontal, then the parhelia become bona fide parhelia.) Similarly, each of the curved segments in Wegener's tangent arc is a rotated parheliion. The parhelia come in pairs, one in the upper tangent arc and one in the lower, with each pair corresponding to a different fixed horizontal direction of the crystal axis. The apparent asymmetry between the two parhelia in each pair is only due to Wegener's choice of projection.

This insight—that column arcs consist of rotated plate arcs—goes all the way back to Huygens. He used it, as just described, to calculate the correct shape of the tangent arc (Figure 3.6), even though his ice particle model was not correct. Using this same insight, Bravais recognized that, just as the tangent arc is the column-analogue of the parhelia, the infralateral arc is the column-analogue of the circumzenith arc (both have ray path 13), and the supralateral arc is the column-analogue of the circumhorizon arc (both have ray path 32). Later Wegener explained the halo now known as the Wegener arc; it is the column-analogue of the subparhelia (both have ray paths 325 and 327). Wegener also recognized that the subhelic arc is the column-analogue of the 120° parhelia. Although the Huygens-Bravais-Wegener insight is the classical approach to understanding column arcs, some care is required to carry it out in detail, as can be seen even in Wegener's diagram.

For column orientations the helioc, subhelic, anthelic, and subanthelic points of the celestial sphere play a special role (Figure 6.5). To see why, imagine again that the crystal axis is fixed (and horizontal) and that the crystal rotates about the axis. The resulting “halos” lie on four vertical circles, just as the halos from plate orientations lie on four horizontal circles. Two of those vertical circles are relevant at the moment. One passes through the helioc point (the sun) and the subhelic point, and the other passes through the anthelic point and the subanthelic point. This is true regardless of the (horizontal) direction of the crystal axis,

	Halo		Ray path
22° column arcs	Tangent arc	T	3 5
46° column arcs	Infralateral arc	INF	13
	Supralateral arc	SUP	31
Arcs involving reflection	Parhelic circle	PHC	1, 316
	Wegener arc	W	315
	Subhelic arc	H_2	1342, 1352
	Anthelic arc		31574, 12351
	Diffuse arcs	D	3157, 325673

FIGURE 6.4 Column arcs from prismatic crystals. The anthelic arc is listed in the table but appears in neither simulation.

FIGURE 6.5 Helic, subhelic, anthelic, and subanthelic points on the celestial sphere.

even though the circles themselves do depend on the direction of the crystal axis. Column orientations therefore have a natural potential to focus light on the helic, subhelic, anthelic, and subanthelic points; near these points many different crystal orientations can work together to light the sky. Whether you actually get light at these points, however, depends on a number of other factors as well.

Arcs from Parry orientations

A crystal is in *Parry orientation* when two prism faces are horizontal. We always assume that in Parry orientation it is face 3 that is the top horizontal face. Crystals with Parry orientations are more severely constrained than are those with column orientations. Whereas a crystal in column orientation is free to rotate both about its own axis and about the vertical axis, a crystal in Parry orientation can only rotate about the vertical axis.

Arcs from Parry orientations are shown in the simulations in Figure 6.6. Although these arcs are rare, they do occur, and they can be spectacular, as in the display shown in Figures 6.7 and 6.8. Most of the halos identified in the $\Sigma = 20^\circ$ simulation can be seen in that display. Arcs from Parry orientations can also be seen in Figure 1.8; we ignored them earlier, in the interests of simplicity.

Arcs from Parry orientations are easier to understand than are column arcs (i.e., halos from column orientations), since, as for plate arcs, the set of responsible crystal orientations is one-dimensional rather than two-dimensional.

Figure 6.9 shows how the upper suncave (22°) Parry arc forms. The responsible ray path is 35; the ray enters the top horizontal prism face and exits a lower sloping prism face. The normal direction to the entry face is therefore vertical, just as it was for the circumzenith arc (Figure 4.7). Therefore, to get the light point **R** of the internal ray, you project the sun point directly upward to the outer sphere. Then to get the halo point, you project **R** back to the inner sphere, with the projection being in the direction normal to the exit face. The normal

Halo			Ray path
22° Parry arcs	Upper suncave	P ₁	35
	Lower suncave	P ₂	46
	Upper sunvex		48
	Lower sunvex		57
46° Parry arcs	Circumzenith arc	CZ	31
	Circumhorizon arc	CH	16
	Infralateral	I46	17, 15
	Supralateral	S46	41, 81
Arcs involving reflection	Parhelic circle	PHC	1, 417
	Hastings arc (of P ₁)	P ₁	315
	Hastings arc (of P ₂)	P ₂	416
	Helic arc	H ₁	4, 5, 152, 345
	Subhelic arc	H ₂	1452
	Anthelic arc		36184, 418765485
	Subanthelic arc	H ₄	3145, 4187654386

FIGURE 6.6 Arcs from prismatic crystals having Parry orientations. Not all of the arcs in the table appear in the simulations.

FIGURE 6.7 Halo display, South Pole, January 11, 1999. The display has halos from plate, column, Parry, and random crystal orientations; nearly all of the labeled halos in the $\Sigma = 20^\circ$ simulations of Figures 6.2, 6.4, and 6.6 appear here. In this display the halos from Parry orientations are the best that we know of. Sun elevation $\Sigma = 22^\circ$. Photo © Marko Riikonen.

FIGURE 6.8 Same display as in Figure 6.7. Arcs here that arise exclusively from Parry orientations are the upper suncave Parry arc, the left supralateral (46°) Parry arc, the helic arc, the subanthelic arc, and the Hastings arc, the latter perhaps being too faint to survive reproduction.

FIGURE 6.9 Light point diagram for the upper suncave Parry arc, showing the formation of the arc. The right-hand diagram emphasizes that the halo is on the intersection of a cone with the celestial sphere. Compare with Figure 4.7.

FIGURE 6.10 Light point diagram for the upper sunvex Parry arc. The light points of the internal rays are on the outer sphere as usual, but only the inner sphere is shown here.

to the exit face, instead of being horizontal as it was for the circumzenith arc, is now inclined 30° to the horizontal. Thus the segments emanating from **R** in the figure make up part of a cone. The halo is on the intersection of the cone with the celestial sphere.

Figure 6.10 shows the formation of the upper sunvex Parry arc. The ray path is 48; the ray enters and exits the upper sloping prism faces. As usual, you get the halo point by projecting the sun point **S** to the outer sphere and then back to the inner sphere. For each crystal orientation the two projection directions are inclined 30° to the horizontal and are in a vertical plane passing through **S**.

It is clear from the previous example—the upper sunvex Parry arc—that the circumzenith arc (Figure 4.7) and the upper suncave Parry arc (Figure 6.9) owe their simplicity to a kind of degeneracy, and that they are not representative of halos in general. But even the upper sunvex Parry arc has special properties that

FIGURE 6.11 (Left) Light point diagram for the helic arc. The arc is the intersection of the celestial sphere with a cone whose vertex is at the sun. (Right) Same but also showing the subanthelic arc, which is the intersection of the sphere with a cone whose vertex is at the subanthelic point.

make it relatively simple and not entirely representative. Fortunately for the early halo theorists, most of the classical halos are special in one way or another, and they are therefore simpler to understand than they might have been. Of the halos arising in Parry orientations, the ones that are least special are the infralateral and supralateral 46° Parry arcs (**I46** and **S46** in Figure 6.6), but they were virtually unknown until recently. Although it is easy enough to instruct the computer to draw the analogues of Figure 6.9 for these arcs, the resulting diagrams are not all that helpful, and we omit them.

Far simpler to understand are the four closely related helic, subhelic, anthelic (or Tricker), and subanthelic arcs. The simplest is the helic arc, and it is simple indeed (Figure 6.11). The responsible ray path is essentially a single reflection off a sloping prism face. Only a single sphere, of radius one, is required to depict the formation of the halo, and all of the projections go from this sphere to itself. Each halo point is found by projecting the sun point in the direction normal to the reflecting face. Since the crystal is in Parry orientation, the normal direction is inclined 30° to the horizontal plane. As the crystal rotates about the vertical, the normal directions sweep out a vertical cone with vertex at the sun and with a vertex half-angle of 60° . The helic arc is the intersection of the cone with the celestial sphere.

The subanthelic arc, also shown in the figure, is the intersection of another cone with the celestial sphere, but it is harder to see why. The cone turns out to have vertex at the subanthelic point and to have a vertex half-angle of 30° . For the subhelic and anthelic arcs, you reflect this figure in the horizontal plane.

$\Sigma = 20^{\circ}$

Halo		Ray path
22° Lowitz arcs	Upper	35
	Lower	46
	Unnamed	48
46° Lowitz arcs	"	81
	"	31
	"	41
	"	14
	"	15
Arcs involving reflection		" 4

FIGURE 6.12 Arcs from prismatic crystals having Lowitz orientations. The 46° Lowitz arcs with ray paths 14 and 15 are long arcs that are difficult to pick out in the simulation; they are not labeled. Sun elevation $\Sigma = 20^{\circ}$. The crystal spins about the indicated horizontal axis.

Arcs from Lowitz orientations

Arcs from Lowitz orientations are rare. We mention them here partly because Lowitz arcs are part of halo lore, and partly because having available this additional class of crystal orientations (Lowitz orientations) puts the other crystal orientation classes in better perspective.

A crystal is in *Lowitz orientation* when a long axis of a basal face is horizontal. Physically, the crystal is supposed to be spinning about this horizontal axis, or rather about a parallel axis through the center of the crystal.

Arcs from Lowitz orientations are shown in Figure 6.12. The nomenclature for these arcs is not settled, and in the simulation we have just labeled the arcs with their ray paths. The arc 46 is presumably the arc seen by Tobias Lowitz in the famous St. Petersburg display of 1790; Lowitz [43] described it as a short colored arc extending from the parhelion downward towards the 22° halo. Lowitz did not, however, report any of the other arcs that might be expected from Lowitz orientations.

In the past we expressed doubts about the existence of Lowitz arcs. We were wrong, and in the past decade a number of photographs have been taken that show 22° Lowitz arcs.

Some of these photos have been analyzed by Riikonen et al [60]. They found that while a simple spinning crystal like that of Figure 6.12 would simulate the shapes of the halos correctly, it failed to get the intensities right. To get reasonable intensities they had to adjust both the shapes and orientations of the crystals.

The circular halos

More or less randomly oriented prismatic crystals make the 22° and 46° circular halos. The orientations need not be completely random, and in fact truly random orientations may be the exception rather than the rule. But to avoid repeated qualifications, we will nevertheless speak of the circular halos as arising in randomly oriented crystals. Circular halos are treated in detail in Chapter 8.

Sun pillar

The sun pillar is a vertical column of light that seemingly emanates from the rising or setting sun (e.g., Figure 18.3, top). It is due to reflections of sunlight in nearly horizontal crystal faces. Most often the crystals are oriented hexagonal plates, but other shapes and orientations are also possible; all that is required is low sun and a preponderance of nearly horizontal crystal faces.

The sun pillar is anomalous in the following sense: Most (non-circular) halos are best when the tilts of the crystals are nearly zero. But the sun pillar disappears when the tilts are zero. (Or it becomes the subsun, depending on your point of view.) In this sense the sun pillar does not have a halo identity of its own.

Odd Radius Halos Are Real

Confusion and doubts

Hexagonal prismatic crystals—essentially the model of Mariotte—can account for many halos quite well, but they cannot explain the odd radius display of Helmut Weickmann, in which several circular halos have radii other than the usual 22° and 46° (Figure 1.1). Weickmann's display is not unique, and even as early as the seventeenth century there were occasional rumors of odd radius halos. Later, as halo observations continued to accumulate in the eighteenth and nineteenth centuries, a persistent small fraction of them would report odd radius halos. There was little consensus, however, regarding the observed radii of the halos. In fact, there was a nearly continuous range of reported radii running all the way from about 6° up to 28° , with scattered larger radii as well. Perhaps nobody believed all of the reports to be accurate, but nobody could say for sure which ones to reject. Thus the reported radii were difficult to evaluate. Photographs of odd radius halos, which could have confirmed the existence of the halos and given reliable radius estimates, were not forthcoming until recently. On the subject of odd radius halos the observational record was equivocal at best.

It turns out that most odd radius halos are caused by ice crystals having pyramid faces (Figure 7.1). Such crystals are called pyramidal (or pyramid) crystals, whether or not they also have prism and basal faces. Pyramidal crystals seem to occur only under cold conditions, and it is unlikely that any of the early halo workers, working in, say, Paris, would have seen one. But already in the seventeenth century Mariotte had recognized that pyramidal crystals were a theoretical possibility, and he had considered and then rejected them as the

FIGURE 7.1 Some conceivable pyramidal crystals. The left-hand crystal has two basal faces, six prism faces, and twelve pyramid faces. The middle crystal is the same but with no prism faces, and the right hand crystal is the same but with neither prism nor basal faces. We cannot recall seeing a naturally occurring pyramidal ice crystal with no basal faces.

cause of the 22° halo.¹ Much later—in the mid-nineteenth century—Bravais used pyramidal crystals to explain odd radius halos. By then, Scoresby [66] had made some very explicit drawings and descriptions of pyramidal crystals that he had seen in the arctic. Although Scoresby’s crystals may have been bullet crystals (Figure 2.7) rather than true pyramidal crystals, in one respect it did not matter: halo theorists, including Bravais, could believe that pyramidal crystals were real, at least for the time being. Still, the exact shape of the crystals—in particular, the crucial inclination angle between the crystal axis and the pyramid faces—was not evident from Scoresby’s observations. But about the same time as Scoresby, Edward Daniel Clarke [12] reported seeing rhombohedral ice crystals. The crystals were growing on bridge timbers and had apparently formed in mist from a river. They were huge—about an inch long—and the crystal faces were good enough that the interfacial angles could be measured with a goniometer. Rhombohedral crystals should in some ways not be so different from pyramidal crystals, and Bravais used Clarke’s results to infer the likely shape of pyramidal crystals. Although Clarke’s results influenced Bravais and others after him, even well into the twentieth century, we—Tape and Moilanen—do not know what to make of Clarke’s observation; we ourselves have never seen anything quite like it. In any case, when photographs of ice crystals began to appear in the literature in the late nineteenth and early twentieth centuries, no pyramidal or rhombohedral crystals were to be seen. The observational record of pyramidal crystals was as murky as that of odd radius halos.

The ambiguous observational record, both of odd radius halos and of pyramidal crystals, made for a poor foundation for the theory of odd radius halos, and the

¹ Mariotte even claimed to have seen some pyramidal crystals, but his claim seems to have been forgotten, deservedly so.

theory was for a long time, well, a mess. As a result, even as late as the 1970's one sometimes heard doubts about the existence of odd radius halos. But by that time there were emerging signs of order amidst the chaos, and more such signs were on the way.

Making some headway

The crystallographic ratio c/a of ice, to be explained later, is a crucial parameter for halo theory. Together with the hexagonal symmetry of ice, the c/a ratio determines which crystal faces are likely to occur. What it determines is not the relative sizes of the faces but rather their relative orientations, hence the interfacial angles. The interfacial angles in turn determine the radii of any resulting circular halos. In short, the c/a ratio determines the likely halo radii.

The c/a ratio of ice was not reliably known until the 1920's, when William Howard Barnes [3] and others measured it using X-ray diffraction techniques. But from then on, halo theorists could in principle predict the radii of the most likely odd radius circular halos. Steinmetz and Weickmann did so, in 1947 in their article *Zusammenhänge zwischen einer seltenen Haloerscheinung und der Gestalt der Eiskristalle* [72],² in which they analyzed Weickmann's odd radius halo display of 1940. They assumed that the responsible ice crystals had the simplest crystallographically possible pyramid faces, the so-called $\{10\bar{1}1\}$ faces (Chapter 9). Using Barnes' c/a -value, they calculated the interfacial angles on the crystals, and then they used the interfacial angles to calculate the expected halo radii. Their calculated radii agreed passably well with the crude observational estimates of radii that Weickmann had made during the display. In the next two chapters we will essentially repeat the calculations of Steinmetz and Weickmann.

Weickmann had not been able to get a good look at the crystals during the display, but he and Steinmetz did see pyramidal crystals at other times. The crystals were not bullets but had good planar faces—the crystals were truly pyramidal. Some, at least, of the crystals had shapes that were similar to those of the hypothetical crystals that they had used to explain the halo display—the pyramid faces appeared to be the $\{10\bar{1}1\}$ faces.

Still missing was any photographic documentation of odd radius halos, but it came eventually. Turner and Radke [80] in 1975 and then Tricker [79] in 1979 published unequivocal photographs of odd radius halos. Somewhat later Neiman [50] published photographs of a spectacular and long lasting odd radius display in Boulder, Colorado. Today we have hundreds of good photographs of odd radius halo displays. Odd radius halos are real, and so are the pyramidal crystals that make them.

² The Relation between a Rare Halo Phenomenon and the Form of the Ice Crystals

Most of the measurements of halo radii that have been made from photographs are consistent with the values predicted by Steinmetz and Weickmann. But the measurements are often imprecise, for a variety of reasons, and we in fact need more good measurements of halo radii. For those who wish to measure halo radii from their own photographs, Appendix C has some suggestions for taking the photographs.

In hindsight we tend to see the Steinmetz and Weickmann article as a turning point in our understanding of odd radius halos. But their theory was not immediately embraced. It was not widely known, especially in the English speaking world, and where it was known, it did not get much attention. Perhaps some skepticism was justified initially, given the lack of photographs of odd radius halos at the time. In any case, the historical evolution of our understanding of odd radius halos is more convoluted and tortuous than the above summary suggests. In Chapter 11 we will take another look at it.

How rare are odd radius halos?

Since good documentation of odd radius halos was so long in coming, you might think that odd radius halos are impossibly rare. Indeed they are rare, at least in most geographic locations, but not as rare as might be imagined. One of the odd radius circular halos—the 9° halo—is close to the sun and therefore easily missed, even when it is fairly bright. Two of the odd radius circular halos—the 23° and 24° halos—are often difficult to distinguish from the ordinary 22° halo, and the 18° and 20° halos are often difficult to distinguish from each other. And, like any halos, the odd radius halos are easy to miss when faint. When halos are present we recommend scanning the sky using a \$1.59 circular rearview mirror, the sort that is sold in many auto supply stores under the name blind-spot mirror. The mirror compresses the halos and dramatically increases contrast, thus increasing the chances of detecting faint halos. Photographs of displays can also be scrutinized for faint halos; we are still embarrassed on occasion to find halos in our photos that we did not see when the display was in progress. Finally, practice helps enormously. The dentist sees more in the dental X-ray than does the patient, the geologist sees more in the rock outcrop than does the passing hiker, and the seasoned halo observer will see more in the sky than does the novice, even though both are looking at the same sky. Still, in most localities even an experienced and persistent observer will be lucky to see more than a handful of odd radius halo displays per year.

How rare are pyramidal crystals?

Odd radius halos are normally an indication of pyramidal crystals in the atmosphere, but it does not work the other way around. That is, you can have pyramidal crystals without having odd radius halos. Pyramidal crystals are therefore more common than you would guess from the frequency of odd radius halo displays. In our ice crystal samples collected at temperatures below about -20°F we often see a few pyramidal crystals, though usually not enough to make perceptible halos. Our Fairbanks, Alaska, crystal samples collected at temperatures below about -30°F are often dominated by pyramidal crystals (e.g., Figure 10.14), but below about -35°F the crystals are usually too small to make good halos. These remarks are an oversimplification, and we would not want to have to make a living predicting crystals or halos on the basis of temperature (or anything else).³ Nevertheless, our experience is that pyramidal crystals require cold conditions.

Reports of pyramidal crystals are nearly non-existent in the many published studies of atmospheric ice crystals. One reason that few scientists have seen pyramidal crystals is that they were working in relatively warm places, especially Japan. But lots of crystals have been collected from aircraft flying through cold clouds, lots of crystals have been collected at the ground in cold places, and lots of crystals have been grown under cold conditions in laboratories. We really do not understand why pyramidal crystals are not being seen more often in these studies.

You will probably never see a pyramidal ice crystal in, say, St. Louis, since it is not cold enough there. But there can be pyramidal ice crystals nearby—within ten miles of downtown—even in July. They are straight up, of course, and when there are enough sufficiently large ones, you will see their characteristic signature in the sky—the odd radius halos.

³ Indoors in the laboratory the dependence of crystal habit on temperature and humidity is relatively well defined, though complex [2]. Outdoors any such dependence is much more elusive.

FIGURE 8.1 Odd radius circular halos seen from an airplane at 31,000 feet over British Columbia. In the photograph the 18°, 22° (or 23°?), and 35° halos are colorful and easy to see. Between the 18° and 22° halos the 20° halo is faint but unusually sharp and distinct. The 23° and 24° halos are probably present but masked by the bright 22° halo. The 9° halo is bright but nearly lost in the glare at the top of the airplane window. The simulation of the display was made using randomly oriented crystals shaped like those shown. The pyramidal crystals made the odd radius halos in the simulation, and the prismatic crystals made the 22° halo. The pyramid faces of the crystals are the $\{10\bar{1}1\}$ faces, the simplest crystallographically possible pyramid faces. Tick marks in the simulation are at intervals of one degree.

Odd Radius Circular Halos

The halo display in Figure 8.1 was seen from a commercial jet flying over British Columbia between Seattle and Anchorage in 1999. The display consists of several odd radius circular halos and in fact is probably almost identical to the display seen by Weickmann nearly six decades earlier (Chapter 1). The simulation in the figure shows the circular halos that would be theoretically expected from randomly oriented prismatic and pyramidal crystals as shown. The pyramid faces of the crystals are the $\{10\bar{1}1\}$ faces, which, as mentioned in Chapter 7, are the simplest theoretically possible pyramid faces. The pyramidal crystals here can therefore be regarded as the next logical step in complexity after the simple prismatic crystals of Chapters 1–6. Using the same ideas that guided Steinmetz and Weickmann (Chapter 7), we will see that crystals like these should theoretically be able to make as many as eight circular halos, having radii of about 9° , 18° , 20° , 22° , 23° , 24° , 35° , and 46° . The radii of the halos in the photograph are difficult to measure accurately, but they appear to be at least roughly consistent with the theoretically expected values. (Although only five halos are seen distinctly in this particular photograph, the 22° , 23° , and 24° halos are probably overlapping to give the appearance of a single halo. The 46° halo, though seen only faintly here, is stronger in other photographs of the display.) These eight halos are by far the most common of the circular halos; any other circular halo would be considered highly exotic.

The circular halos arise in (more or less) randomly oriented crystals. Their ray paths enter and directly exit the crystal, with internal reflections playing no essential role. Each circular halo is associated with such a ray path or, equivalently, with a *wedge angle*, the wedge angle being the angle between the entry and exit faces of the ray path. Table 8.1 gives the wedge angles and theoretical halo radii

FIGURE 8.2 Face numbering on a pyramidal crystal. The basal faces are numbered 1 and 2, and the prism faces are 3, 4, . . . , 8, the same as for prismatic crystals. The pyramid faces are 13, 14, . . . , 18, and 23, 24, . . . , 28. The pyramid faces here are the {10 $\bar{1}$ 1} faces, but the numbering scheme is standard for any pyramidal crystal.

for the eight circular halos mentioned above.

How do the theoretical halo radii in the table square with the radii of real halos? The British Columbia display is not a good test, because, as already mentioned, the halo radii in the photo are hard to measure. Not only is the sun out of the photo, there is also the complication of distortion introduced by the airplane window.

The display shown in Figures 8.3 and 8.4 has fewer halos, but it is a very strong display nevertheless. Using the methods of Appendix D, we have positioned orange, blue, and red dots in the photo at respective angular distances of 18.3°, 22.9°, and 34.9° from the sun; these are the theoretical radii of the 18°, 23°, and 35° halos (Table 8.1). The orange, blue, and red dots do indeed end up on the inner edges of three circular halos; apparently the halos are the 18°, 23°, and 35° halos. The 20° halo may be present as well, blending with the 18° halo. But although ordinary 22° parhelia are prominent, there is no 22° halo. That is, there is no circular halo at the green dots, which are 21.8° from the sun, where the 22° halo would be

TABLE 8.1 Wedges (or ray paths), wedge angles, and theoretical halo radii of circular halos made by ice crystals having basal, prism, and {10 $\bar{1}$ 1} pyramid faces (Figure 8.2).

Wedge	Wedge angle α	Halo radius $\Delta_{\min}$	Halo name
13 6	28.0°	8.9°	9° halo
13 25	52.4	18.3	18° halo
13 16	56.0	19.9	20° halo
3 5	60.0	21.8	22° halo
13 2	62.0	22.9	23° halo
13 5	63.8	23.8	24° halo
13 15	80.2	34.9	35° halo
1 3	90.0	45.7	46° halo

FIGURE 8.3 The 18°, 23°, and 35° halos. Ordinary 22° parhelia are present as well, at left and right, clearly closer to the sun than is the 23° halo. The colored dots are at the indicated angular distances from the sun. The sun, barely more than a grey dot here, is seen through a nearly opaque disk made from crossed polarizers. Fairbanks, January 31, 2005.

FIGURE 8.4 Slightly later stage of the same display, photographed with a wider angle lens and different film.

FIGURE 8.5 Odd radius lunar halos. The indicated angular distances from the moon to the circled stars can be used to estimate the halo radii. The most prominent halos appear to be the 9°, 18°, and 23° halos. Fairbanks, Alaska, January 5, 2004, 10:46 P.M. Moon elevation 50°.

expected to be. The significance of the yellow dot, at the 1:30 position and 32° from the sun, will be explained in Chapter 11, in connection with Figure 11.4.

Positioning the colored dots at prescribed angular distances from the sun is more involved than it may sound, and measuring the radii of daytime halos from a photograph is always a tricky business. But a photograph of lunar halos is another matter. If the photo contains some stars, then accurate angular distances from the moon to the various stars are easily computed, so long as the time and place of the photo are known.

Thus, in Figure 8.5 the angular distances from the circled stars to the moon are known quite accurately—to better than .05°. The positions of the circled stars relative to the moon and the halos are just what you would expect from Table 8.1 if the halos in the photo are the 9°, 18°, and 23° circular halos: The position of Saturn, 9.3° from the moon and about at the inner edge of the inner halo, is just right if the halo is the 9° halo (theoretical radius 8.9°). The position of the star δ Gem at 18.1° from the moon and at the inner edge of the intermediate halo, is correct if the halo is the 18° halo (theoretical radius 18.3°). The positions of the three outer stars suggest that the outer halo is the 23° halo rather than the common 22° halo; the halo is just a bit too far from the moon to be the 22° halo.

FIGURE 8.6 Idealized simulation of the 22° halo. The crystals used in making the simulation were randomly oriented hexagonal prisms. Only ray paths 35 were allowed, so that, for example, there were no reflected rays and no 46° halo rays. The two circles are the theoretical inner and outer boundaries of the halo, on which $\Delta = \Delta_{\min}$ and $\Delta = \Delta_{\max}$. The sun was a point sun, and the light rays were all of one wavelength; as a result, the inner edge of the halo appears unrealistically sharp.

In the next chapter we will see how the wedge angles in Table 8.1 were calculated. In doing so, we will also begin to see in what sense the $\{10\bar{1}1\}$ faces are simple. In the next two sections of the present chapter we will see how the halo radii in Table 8.1 were calculated from the wedge angles. We begin by examining the 22° halo; it will serve as a prototype for all circular halos.

The 22° halo as a prototype for all circular halos

The 22° halo normally arises in prismatic crystals, but it can also arise in pyramidal crystals like the one in Figure 8.2, since that crystal has the necessary prism faces. The crystals should be randomly oriented and the ray path should be 35 (or 46, 57, etc.), so that the wedge angle is 60°. As a crystal takes on all orientations, the halo point—the light point of the outgoing ray—traces out the halo. Since all orientations are allowed, you might think that the halo point would trace out the entire celestial sphere, but in fact the angular deviation Δ between the sun and the halo point ranges not from 0° to 180° but only from $\Delta_{\min} = 21.8^\circ$ to $\Delta_{\max} = 50.1^\circ$, and the halo point traces out an annular region on the celestial sphere (Figure 8.6). The portion of the halo near the outer boundary $\Delta = \Delta_{\max}$ is far too weak to be seen, and normally what you notice in the sky is mainly the relatively abrupt change in brightness at the inner boundary $\Delta = \Delta_{\min}$. In fact we often carelessly refer to the inner boundary as the 22° halo, thus treating the halo as a circle rather than as an annular region.

Figure 8.7 illustrates the wedge angle for the 22° halo. The figure also attempts to illustrate the idealized wedge that is the source of the term wedge angle. The wedge is bounded by planar extensions of faces 3 and 5 and extends indefinitely

FIGURE 8.7 (Left) Wedge angle $\alpha = 60^\circ$ for the wedge 35 on a crystal as in Figure 6.1 or Figure 8.2. This wedge gives the 22° halo. (Right) Wedge 35. The wedge is determined by faces 3 and 5 of the crystal. The hexagon represents the crystal, and the large triangle enclosing the hexagon represents the wedge. The view is looking directly at face 2 of the crystal.

in both directions perpendicular to the page, and indefinitely in the direction toward the bottom of the page as well, so that there are only two faces bounding the wedge. We sometimes imagine for simplicity that the 22° halo arises in wedges like this one.

Halo radii from wedge angles

Other circular halos are like the 22° halo but with different wedge angles α . Both $\Delta_{\min}$ and $\Delta_{\max}$ depend on α and, hence, so does the annular region occupied by the halo. The parameter $\Delta_{\max}$ is usually unimportant, since the halo is so weak near its outer boundary, but $\Delta_{\min}$ is crucial—it is the halo radius. In the next section we will derive the classical expression for the dependence of $\Delta_{\min}$ on α , namely,

$$\sin \frac{\alpha + \Delta_{\min}}{2} = n \sin \frac{\alpha}{2} \quad (8.1)$$

where $n = 1.31$ is the refractive index for ice. For the 22° halo, the wedge angle α is 60° and so $\Delta_{\min}$ is 21.8° . For the 46° halo, the wedge angle is 90° and $\Delta_{\min}$ is 45.7° . The other halo radii $\Delta_{\min}$ in Table 8.1 are calculated similarly, once their wedge angles are known.

In general, larger wedge angles give larger halo radii. However, there is a largest wedge angle $\alpha_{\max}$ that will allow light to pass through the wedge. For a wedge with $\alpha > \alpha_{\max}$, any light ray that enters the wedge at one face suffers total internal reflection at the other. We will see that $\alpha_{\max}$ is given by

$$n \sin \frac{\alpha_{\max}}{2} = 1 \quad (8.2)$$

For ice, with $n = 1.31$, Eq. (8.2) gives $\alpha_{\max} = 99.5^\circ$. Equation (8.1) then gives a largest possible halo radius of $\Delta_{\min} = 80.5^\circ$, but in practice this value is too generous, and any halo having a radius greater than about 50° would probably be too faint to be seen.

On a hexagonal prism there are three wedge angles, namely, 60° , 90° , and 120° . The wedge angles 60° and 90° make the 22° and 46° halos, but the wedge angle 120° is greater than $\alpha_{\max}$. The 22° and 46° halos are therefore the only circular halos that can arise in hexagonal prisms. To make additional circular halos, you need additional wedge angles and therefore a crystal other than a simple hexagonal prism. A pyramidal crystal is a natural choice, but to specify its shape—and hence the wedge angles—you need to know the angle of inclination between the pyramid faces and the crystal axis, and this is where halo theorists were for a long time in the dark. Had they known this angle, they could have found the wedge angles and then calculated the halo radii using Eq. (8.1), which has long been known. On the other hand, had they known the halo radii with some precision, they could have worked backwards to get the wedge angles and perhaps the crystal shape. But they knew neither, at least not with any certainty.

Today we have many photographs of odd radius halos, and we can measure the halo radii from the photos. The measurements require care, however, and even under the best circumstances the measured radii are uncertain by half a degree or so, the uncertainty being due mainly to the fuzziness of the halo edge. We also have photographs of pyramidal crystals, and we can measure wedge angles from the photos, but again the results are not as precise as we would like. The most precise values for wedge angles are today calculated theoretically from crystallographic principles. In the next chapter we will show how these calculations go.

Topics in the present chapter have been arranged roughly in order of decreasing importance. On a first reading, you may wish to skip directly from here to the next chapter.

How light passes through a wedge

In order to understand circular halos and, in particular, in order to derive Eq. (8.1), we need to think about the way light passes through a wedge. How much is the light deviated? Is there a minimum value for the deviation? Is there a maximum? And so forth. The problem of light passing through a wedge is not so simple, since it is spatial. Veterans of Physics 101 may remember something like it, but usually what is treated there is the special case where the ray path lies entirely in a plane normal to both faces of the wedge. In many books on halos the problem is similarly swept under the rug. The general problem, not just the special case,

has been treated by Uhler [81] and Tape [75], both of whom approached the problem analytically. Our approach here is geometric. You will see the appeal of the geometric approach if you glance at one of the analytic treatments for comparison.

As usual, we consider a light ray passing through a wedge having refractive index n and wedge angle α . Figure 8.8 shows the light points **S**, **R**, and **H** of the entry, internal, and exit rays. The sun point **S** and the halo point **H** are on the sphere of radius 1, while **R** is on the sphere of radius $n > 1$. The law of refraction (page 38) tells how to get **H** from **S**: you project **S** to the outer sphere, getting **R**, and then project **R** back to the inner sphere, getting **H**. The first projection is normal to the entry face, the second is normal to the exit face.

We refer to the V-shaped configuration consisting of the two red line segments **RS** and **RH** as the *vee* or the *normal vee*,¹ since its legs—the two line segments—are the projection directions and hence are normal to their respective wedge faces. As the wedge takes on various orientations, the point **S** in Figure 8.8 stays fixed, but **R** moves on the outer sphere and **H** moves on the inner sphere, always in such a way that the vertex angle of the vee remains equal to the wedge angle α . The halo point **H** traces out the circular halo.

Minimum deviation In Figure 8.8 the deviation between the points **S** and **H** is, in radians, the great circle distance between **S** and **H**. It is closely related to the straight line distance between **S** and **H**; the smaller the deviation, the smaller the straight line distance. It is clear from the figure that you can't make **S** and **H** arbitrarily close together and you can't make them arbitrarily far apart. So you can't make the deviation arbitrarily small and you can't make it arbitrarily big. What are the normal vees for which the deviation is minimum? What are those for which it is maximum?

Figure 8.9 shows how to narrow down the candidates both for minimum and for maximum deviation. If you start with a normal vee that is not isosceles, you can always find a nearby vee which has smaller deviation: In the top diagram, if you start with the vee formed by **S**, **R**, and **H**, you can perturb it to the vee formed by **S'**, **R** and **H'**, always keeping the vertex angle equal to α . Since the segment **RH** was more nearly perpendicular to the inner circle than was **RS**, the perturbation decreases the deviation. (The straight-line distance between **S'** and **H'** is less than that between **S** and **H**.) Continuing this process, you arrive at an isosceles vee (middle diagram) which has smaller deviation than the original vee.

A vee is said to be *central* if the plane of the vee passes through the common

¹ More precisely, points **S**, **R**, and **H** define a *normal vee* (with parameters n and α) if **S** and **H** are on the inner sphere, **R** is on the outer sphere, $\angle SRH = \alpha$, and the line segments **RS** and **RH** do not penetrate the inner sphere. In this case the corresponding ray path and wedge, including the wedge orientation, can be reconstructed from **S**, **R**, and **H**.

FIGURE 8.8 Light point diagram for the circular halo forming in a wedge having refractive index n and wedge angle α . The spheres are concentric and have radii 1 and n . As the wedge (not shown) changes its orientation, the normal vee (red), consisting of the line segments **RS** and **RH**, changes also; the sun point **S** is fixed on the inner sphere, and the vertex angle of the vee remains equal to α , but the point **R** can move on the outer sphere, and the halo point **H** can move on the inner sphere, where it traces out the halo.

center **O** of the two spheres. Among all the isosceles vees **SRH** with vertex at **R** and with fixed vertex angle α , the ones with smallest deviation are clearly those that are central. After all, the isosceles triangles **SRH** are all similar, and we just want the ones with the shortest side **SH**.

Thus, among all vees associated with the given wedge, the ones that give minimum deviation $\Delta_{\min}$ are those that are isosceles and central. One such vee is shown at the top in Figure 8.10. The Law of Sines applied to the lower left diagram in the figure gives Eq. (8.1), the fundamental equation for finding halo radius in terms of wedge angle.

Historically, any plane that is normal to both faces of the wedge has been known as a *normal plane*. A vee is central when the corresponding ray path lies entirely in a normal plane; this is because the plane of any vee **SRH** is itself a normal plane, and because the entry, internal, and exit rays are in the directions **SO**, **RO**, and **HO**, respectively. A ray path has minimum deviation when it lies in a normal plane and is symmetric (Figure 8.10, lower right).

Maximum deviation Figure 8.9 shows that the maximum deviation $\Delta_{\max}$ can only occur when the vee is tangential, that is, when at least one of its legs is tangent to the sphere. It turns out that, depending on n and α , the maximum occurs either when the vee is tangential and central, or when the vee is tangential and isosceles. For ice, with $n = 1.31$, the former configuration gives the maximum deviation if $\alpha \leq 46.9^\circ$, and the latter gives it if $\alpha \geq 46.9^\circ$. The result is

$$\Delta_{\max} = \begin{cases} -\alpha + \cos^{-1}(\cos \alpha - \sqrt{n^2 - 1} \sin \alpha) & \text{if } \alpha \leq 46.9^\circ \\ 2 \sin^{-1}(\sqrt{n^2 - 1} \sin \frac{\alpha}{2}) & \text{if } \alpha \geq 46.9^\circ \end{cases} \quad (8.3)$$

For example, the outer boundary of the 22° halo is at $\Delta_{\max} = 50.1^\circ$, since $\alpha = 60^\circ > 46.9^\circ$. For more on both minimum and maximum deviation see the article by Tape [74].

FIGURE 8.9 Several normal vees, all in the same plane, showing that isosceles vees are the only candidates for minimum deviation, and tangential vees are the only candidates for maximum deviation. The situation is similar to that in Figure 8.8 except that here R is fixed, rather than S . At the left in each diagram the points S and H are on the sphere of radius 1. The point R is on the sphere of radius n (not shown). The plane of the vee intersects the sphere of radius 1 in the circle C_1 (red). At the right is the section in the plane of the vee, with circle C_1 as before and with circle C_2 being the intersection of the plane of the vee with the sphere of radius n . Note that the point O' , which is the common center of the two circles, is not the center of the sphere. (Top) An arbitrary vee. Also shown is a perturbation of the vee (dashed) having the same vertex angle and lying in the same plane. Perturbing the vee so as to make it more nearly isosceles, as here, decreases the deviation. Perturbing it so as to make it more nearly tangential increases the deviation. (Middle) Isosceles vee. (Bottom) Tangential vee.

FIGURE 8.10 Calculating the minimum deviation $\Delta_{\min}$ [Eq. (8.1)]. (*Top left*) Light points **S**, **R**, **H** in minimum deviation configuration. The vee formed by **S**, **R**, and **H** is isosceles and central. (*Top right*) Same but showing only the section in the plane of the vee. Since the vee is central, the plane contains the center **O** of the sphere. (*Bottom left*) The triangle **SRO**, which gives Eq. (8.1) via the Law of Sines. (*Bottom right*) The corresponding wedge and ray path. The entry, internal, and exit rays are in the directions **SO**, **RO**, and **HO**, respectively. The ray path is symmetric and lies in a plane normal to both faces of the wedge.

Maximum wedge angle The wedge angle is maximum when a central isosceles vee is also tangential. Then $\angle OSR$ in Figure 8.10 becomes a right angle, giving Eq. (8.2).

Fine tuning the halo radii

One can argue that the theoretical halo radii $\Delta_{\min}$ in Table 8.1 should be refined. Even if the wedge angles in the table are exactly right—which we assume for the moment—there are many factors that might cause small perturbations in halo radii. Some of the perturbations can be predicted quantitatively, but others are elusive. Let’s again consider the 22° halo as an example.

Dependence of halo radius on sun size We have been treating halos as if the light source were a point, but of course the sun has a positive angular diameter—about half a degree. Each point on the sun’s disk can be thought of as making its own 22° halo, and what we normally call the 22° halo is really a superposition of all these halos (annular regions). A small subtraction δ from our value $\Delta_{\min} = 21.8^\circ$ (Table 8.1) should therefore be made in computing the radius of the 22° halo. You might think at first that δ should be equal to the angular radius of the sun, but Figure 8.11 suggests that it should be less, perhaps 0.15° or so. (We are just

FIGURE 8.11 The effect of the positive angular size of the sun on halo radius. (*Top*) Sun and point **H** at a distance $\Delta_{\min} = 21.8^\circ$ (for 22° halo) from the center of the sun. Arc **AB** is part of the circle of radius $\Delta_{\min}$ centered at the center of the sun. Arc **CD** is part of the circle of radius $\Delta_{\min}$ centered at **H**. Points on the white part of the sun's disk can light the point **H** via 22° halo ray paths, since they are $\Delta_{\min}$ or a bit more from **H**. (*Middle*) Same except that **H** has been replaced by **H'** at a distance $\delta = 0.1^\circ$ to the left of **H**, and arc **CD** has been replaced accordingly by arc **C'D'**. Points on the white part of the sun's disk can light **H'**. (*Bottom*) Same but for $\delta = 0^\circ, 0.15^\circ, 0.2^\circ, 0.25^\circ$. With $\delta = 0.2^\circ$ the white fraction of the sun's disk is perhaps too small to be significant. Thus the correction in halo radius due to the positive angular size of the sun should be 0.15° or so.

guessing.) Halo simulations made both with and without a point sun barely differ from each other, thus suggesting that δ might be even less.

Dependence on temperature and pressure It is clear from Eq. (8.1) that $\Delta_{\min}$ depends on the refractive index n of ice as well as on the wedge angle α . But n depends slightly on temperature and pressure, and hence so does $\Delta_{\min}$. A crude approximation for the dependence of n is

$$n(T, P) \approx 1.31 - 3.82 \times 10^{-5}(T + 3) - .035 \times 10^{-5} P, \quad (8.4)$$

where $n(T, P)$ is the refractive index of ice in air at temperature T (degrees Celsius) and pressure P (hectopascals). The base value $n = 1.31$, which appears in Eq. (8.4) and which is the value that was used in calculating the $\Delta_{\min}$ -values in Table 8.1, is for $T = -3$ and $P = 0$. Using Eqs. (8.1) and (8.4), you can estimate the impact of temperature and pressure changes on halo radius. You will find that

the pressure is negligible but that temperature matters, though barely: the radius of the 22° halo at −55°C is about 0.15° larger than that at −3°C. The temperature here of course refers to the temperature where the ice crystals are located, so −55°C is not unreasonable when the halos are forming in high cloud. For such halos the increase in halo radius due to the low temperature should therefore about balance the decrease due to positive sun size. For a halo display forming at warmer temperatures, say −25°C, the change due to temperature would be less, and the temperature and sun size corrections together would give a net decrease of about 0.1° in the theoretical halo radius—not much.

Dependence on wavelength The refractive index also depends on the wavelength (color) of the light under consideration, and hence so does $\Delta_{\min}$. Both n and $\Delta_{\min}$ are smallest for red light, largest for violet. Thus the 22° halo is a superposition of many concentric circular halos, one for each wavelength, with the red halos being the smallest and the violet halos being the largest. The resulting superposition tends to look reddish on its inner edge; this is where only the red halos are seen, with no overlap from the other colors.

Table 8.2 gives $\Delta_{\min}$ both for red light ($n = 1.307$) and for yellow light ($n = 1.31$). When we speak of “the” value of $\Delta_{\min}$ for a halo, we generally mean $\Delta_{\min}$ for yellow light; this is what was given in Table 8.1. But if you are interested in the red, i.e., innermost, part of a halo, then you should use the slightly smaller $\Delta_{\min}$ -value for red light, especially in the case of the larger halos, where the difference between red and yellow becomes substantial.

Are the theoretical halo radii correct? Using the methods of Appendix D, we have positioned the four yellow dots in Figure 8.12 so that their angular distances

TABLE 8.2 Same as Table 8.1 but with halo radii computed for red as well as for yellow light. The 77° halo (bottom line of the table), which in principle exists for red light but not for yellow, would be far too weak to be seen.

Wedge	Wedge angle α	Halo radius $\Delta_{\min}$ (red, $n = 1.307$)	Halo radius $\Delta_{\min}$ (yellow, $n = 1.31$)	Halo name
13 6	28.0°	8.9°	8.9°	9° halo
13 25	52.4	18.1	18.3	18° halo
13 16	56.0	19.7	19.9	20° halo
3 5	60.0	21.6	21.8	22° halo
13 2	62.0	22.6	22.9	23° halo
13 5	63.8	23.6	23.8	24° halo
13 15	80.2	34.5	34.9	35° halo
1 3	90.0	45.1	45.7	46° halo
13 24	99.8	76.9	...	77° halo

from the sun are 21.8° ; this is the $\Delta_{\min}$ -value of the 22° halo for yellow light. The red dots are at 21.6° , the $\Delta_{\min}$ -value for red light. The locations of the dots look about right with respect to the halo, with the red dots ending up on the inner edge of the halo and the yellow dots being just a bit farther out. Here no corrections would be expected for sun size and temperature, since the display occurred in high cloud, where the temperature was probably low, so that the two corrections would cancel each other.

Figure 8.13 is another example, the yellow and red dots having the same meaning as before. To our eye the red and yellow dots in the 12:00 position look about right, but the ones at 3:00 and 9:00 look just a hair too close to the sun. (The ones at 6:00 are in an overexposed region and are irrelevant.) Correcting for sun size and temperature would make the discrepancy worse, though ever so slightly. Perhaps we have a bug in our measurement of angular distances, or perhaps we are still forgetting some adjustment to n . (Or perhaps the whole theory is fundamentally flawed, though of course we do not think so.) But it may also be that the 22° halo here is in fact not perfectly circular. It is easy to see how this might happen if the crystals are not required to have completely random orientations.

In any case, if Figures 8.12 and 8.13 are any indication, the $\Delta_{\min}$ -values in Table 8.1 need very little refinement, if any. If desired, small corrections can be made for sun size, temperature, and color. But usually we will not do so; when we speak of the theoretical radius of a halo, we generally mean the $\Delta_{\min}$ -value from Table 8.1, uncorrected.

The inner edge of a halo is always fuzzy to some extent, which makes it difficult to measure the radius. Because the inner edges of halos are so hard to pinpoint, we have ducked the problem by not attempting to mark them in our photographs. Thus in Figures 8.12, rather than trying to indicate the inner edge of the halo, we have been content to position the red and yellow dots at the angular distances given by the $\Delta_{\min}$ -values for red and yellow light. We then leave it to you to decide whether the edge of the halo is consistent with the dot locations.

Three more caveats Although the analyses in Figures 8.12 and 8.13 are reassuring, we have no reason to be complacent. We can think of at least three more factors that complicate the treatment of halo radii: halo brightness, atmospheric refraction, and diffraction.

In Figure 3.7 the bright lower tangent arc is slightly closer to the moon than is the much dimmer 22° halo—an impossibility, according to theory (Figure 12.2). But that theory ignores the role of the eye and the brain, it ignores the role of the camera and all of the attendant issues involving exposures, processing, and possible electronic manipulation of the photo, and it ignores forward scattering of light, the same forward scattering that creates the aureole around the sun and

FIGURE 8.12 Ordinary 22° halo. Yellow dots are 21.8° from the sun, red dots are at 21.6° (Table 8.2). Fairbanks, July 9, 2004.

FIGURE 8.13 Another display with 22° halo. South Pole, December 19, 1997.

FIGURE 8.14 Odd radius halos forming in very small crystals. Due to diffraction, the halos are so diffuse that their inner edges are impossible to locate with any precision. The halos here are virtually indistinguishable from each other. Fairbanks, January 19, 1996.

makes the sun look far bigger than its half-degree diameter. We can imagine that those things matter, but we have no idea of their relative importance. Anyway, halos look slightly bigger when they are very bright than when they are dim—bigger in the sense that they expand in all directions. That means that the radius of a circular halo will appear to decrease slightly as the halo brightens.

A light ray that is nearly horizontal can sometimes be bent downward slightly by atmospheric refraction, thus raising the light point of the ray by a few tenths of a degree. A ray that is more oblique to the horizon, however, will be nearly straight. The detailed ramifications of this are beyond us, but we can see that atmospheric refraction might introduce an uncertainty of a few tenths of a degree in a halo radius that has one (but not both) of its endpoints near the horizon. One of the endpoints is of course the sun, the other is on the halo.

Finally, diffraction. When the halo-making crystals are small, diffraction broadens the halos and decreases their apparent radii. The decrease can be substantial. (If the crystals are very small, diffraction will wipe out the halos entirely.) We do not know how to incorporate diffraction rigorously into halo simulations. Luckily, halo displays in which diffraction plays a role are easy to spot, due to their diffuse and poorly defined halos (Figure 8.14). Moreover, although diffraction can decrease halo radii, it cannot increase them; an observed

FIGURE 8.15 Odd radius halos. We believe that the halo identifications are correct, but we have not measured the radii. Kustavi, Finland, May 14, 1988. $\Sigma = 44^\circ$. Photo © Pekka Parviainen, Polar Image.

radius that is smaller than $\Delta_{\min}$ can sometimes be attributed to diffraction, but an observed radius that is larger than $\Delta_{\min}$ cannot.

More on the problem of measuring halo radii can be found in an article by Können and Tinbergen [39].

Circular halos are rare

If we insist that circular halos arise in truly random crystal orientations, then circular halos are rare. The halos in the British Columbia display (Figure 8.1) may indeed be circular, but many of the other halos shown in this chapter exhibit intensity variations that are inconsistent with perfectly random orientations. Figures 8.5, 8.15, and 8.16 are examples. Look for weakness or absence of the 18° halo in the 12:00 position, enhancement of the 23° halo in the 12:00 position, or enhancement of the 9° halo in the 6:00 position; these are signs that the crystal orientations have some tendency toward plate orientations rather than being truly random. One can argue about whether the halos in those figures should be called poorly defined plate arcs rather than circular halos. There is no hard and fast line that separates plate arcs (or column arcs) from circular halos. Instead there is a continuum of halos between well-defined plate arcs on one end and truly circular halos on the other.

FIGURE 8.16 Odd radius halos, mainly the 18° and 23° halos. The colored dots are positioned at the indicated angular distances from the sun (Table 8.1). Fairbanks, March 3, 1999. $\Sigma = 12^\circ$.

FIGURE 8.17 Odd radius halos, Vaala, Finland, May 13, 2002. For reasons that are not understood, the 20° halo is not common. Here it is unusually bright and distinct. $\Sigma = 44^\circ$.

Some Crystallography

In the previous chapter we saw how to calculate the radii of circular halos from the wedge angles (interfacial angles) on a crystal. In the present chapter we will see how to calculate the likely wedge angles from crystallographic principles and from the c/a ratio of ice. The crystallographic principles together with the c/a ratio therefore determine the likely halo radii. Not everyone will find this chapter to be easy going. If this applies to you, you can of course skip the work and just take the results on faith. There are two of them. First, on a pyramidal crystal the angle of inclination x of the pyramid faces to the crystal axis determines all of the wedge angles on the crystal. Second, the most likely value of x is 28° . The first statement should seem plausible, perhaps obvious. But the second statement will be hard to swallow—and impossible to appreciate—unless you attempt to follow the reasoning of this chapter.

Wedge angles from angle x

It is easy to calculate the wedge angle of a wedge if the normal vectors to the two wedge faces are known. In Figure 9.1, for example, the wedge angle is the supplement of the angle between the outward normal vectors $\mathbf{N}_3$ and $\mathbf{N}_5$, or, equivalently, it is the angle between $-\mathbf{N}_3$ and $\mathbf{N}_5$.

FIGURE 9.1 Like Figure 8.7 but with the addition of $\mathbf{N}_3$ and $\mathbf{N}_5$, the normal vectors to faces 3 and 5. The wedge angle α is equal to the angle between $-\mathbf{N}_3$ and $\mathbf{N}_5$.

In general,

$$\cos \alpha = -\frac{\mathbf{N}_i \cdot \mathbf{N}_j}{|\mathbf{N}_i| |\mathbf{N}_j|},$$

(9.1)

where α is the wedge angle for entry and exit faces i and j , and where $\mathbf{N}_i$ and $\mathbf{N}_j$ are the outward normal vectors to the two faces.

We said that the inclination angle x determines all wedge angles on a pyramidal crystal. For the wedge 13 14, for example, here is how to calculate its wedge angle in terms of x :

With the crystal axis vertical, as in Figure 9.2, angle x is not only the inclination of the pyramid faces to the crystal axis, it is also the inclination of their normal vectors to the horizontal plane. The outward normal vector $\mathbf{N}_{13}$ to face 13 can therefore be written

$$\mathbf{N}_{13} = (\cos t \cos x, \sin t \cos x, \sin x),$$

(9.2)

where t is the azimuth of the vector. Here we have temporarily assumed the face normals to have unit length.

FIGURE 9.2 (Above) Pyramidal crystal illustrating angle x , that is, the angle of inclination of the pyramid faces to the crystal axis. Angle x determines all wedge angles of the crystal (Table 9.1). Here $x = 39^\circ$, which turns out to be unrealistic.

TABLE 9.1 (Right) Wedge angles α on a pyramidal crystal having inclination angle x (Figure 9.2). For each wedge the table gives $\cos \alpha$ in terms of x . The last seven wedge angles are supplements of the first seven but in reverse order. The wedge angle values in Table 8.1 result from this table by taking $x = 28^\circ$.

Wedge faces	$\cos \alpha$
13 6	$\cos x$
13 25	$(3 - \cos 2x)/4$
13 16	$\cos 2x$
3 5	$1/2$
13 2	$\sin x$
13 5	$(\cos x)/2$
13 15	$(-1 + 3 \cos 2x)/4$
1 3	0
13 24	$-(-1 + 3 \cos 2x)/4$
13 4	$-(\cos x)/2$
13 1	$-\sin x$
3 4	$-1/2$
13 23	$-\cos 2x$
13 14	$-(3 - \cos 2x)/4$
13 3	$-\cos x$

You can get $\mathbf{N}_{14}$ by rotating $\mathbf{N}_{13}$ through an angle of 60° about the vertical axis, that is, you increase its azimuth by 60° , so

$$\mathbf{N}_{14} = (\cos(t + 60) \cos x, \sin(t + 60) \cos x, \sin x) \quad (9.3)$$

Hence from Eqs. (9.1), (9.2), (9.3), the wedge angle α is given by

$$\begin{aligned} \cos \alpha &= -\mathbf{N}_{13} \cdot \mathbf{N}_{14} \\ &= -\frac{1}{2} (\cos^2 x + 2 \sin^2 x) \\ &= \frac{1}{4} (-3 + \cos 2x) \end{aligned} \quad (9.4)$$

The remaining wedge angles are computed similarly and are listed in Table 9.1.

Angle x from crystallography

We now explain how the principles of crystallography limit the possibilities for x . Proofs and more explanation are given in Appendix E.

Ice is a mineral and is therefore expected to conform to the principles of crystallography. A crystal of any mineral has an internal 3-dimensional lattice which, mathematically, consists of all points that are integral linear combinations of three independent vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$. The three vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are said to be a *basis* for the lattice. For minerals like ice that have hexagonal symmetry the lattice and lattice basis are as shown in Figure 9.3; the vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are of length a and make an angle of 120° with each other, and the vector $\mathbf{v}_3$ is of length c and is perpendicular to both $\mathbf{v}_1$ and $\mathbf{v}_2$. The parameters a and c depend on the mineral.

A fundamental tenet of crystallography is that on a crystal of a given mineral only certain crystal faces are possible: For a plane to be a crystal face, it must be a lattice plane, that is, it must contain three non-collinear lattice points. Furthermore, of all the lattice planes, only a small fraction of them are really likely to be crystal faces. How, then, can you recognize a lattice plane, and how can you tell when it is likely to be a crystal face?

Associated with the lattice basis $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ is the so-called dual basis $\mathbf{w}^1, \mathbf{w}^2, \mathbf{w}^3$. Whereas the lattice basis is naturally suited to handle lattice points, the dual basis is suited to handle lattice planes. Any plane not containing the origin has an outward normal vector $\mathbf{N}$ which, like any vector, can be expressed as a linear combination of the dual basis vectors:

$$\mathbf{N} = h\mathbf{w}^1 + k\mathbf{w}^2 + l\mathbf{w}^3 \quad (9.5)$$

FIGURE 9.3 (Top) Lattice basis $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ for ice. The a_1 , a_2 , a_3 , and c -axes are in the directions of $\mathbf{v}_1$, $\mathbf{v}_2$, $-(\mathbf{v}_1 + \mathbf{v}_2)$, and $\mathbf{v}_3$, respectively; thus it is the c -axis, rather than the a_3 -axis, that is in the direction of $\mathbf{v}_3$. (Middle) Same but showing the lattice as well as the lattice basis. The lattice consists of the lattice points (heavy dots), which are understood to extend indefinitely in all directions. (Bottom) The bottom section of the middle diagram but as seen from directly above, looking down the c -axis.

FIGURE 9.4 Lattice planes with Miller indices $1\bar{1}00$, $\bar{1}101$, and $01\bar{1}2$. The $01\bar{1}2$ plane, for example, is parallel to the a_1 -axis and intersects the a_2 , a_3 , and c -axes at $a/1$, $a/(-1)$, and $c/2$, the denominators of these fractions being the Miller indices. In all of Figures 9.3–9.10, the c/a ratio is 1.63; this is the modern c/a value for ice, the same as that found by Barnes (Chapter 7).

for some real numbers h, k, l . If the plane is a lattice plane—and hence a possible crystal face—it turns out that h, k, l are relatively prime integers, that is, whole numbers with no common factors.¹ And although there is a lattice plane for each triple of relatively prime integers h, k, l , that is, there is a lattice plane with normal vector $\mathbf{N}$ given by Eq. (9.5), only the lattice planes with h, k, l small are at all likely to be crystal faces. The integers h, k, l are the *Miller indices* of the face. They are usually written without commas, and with negative integers indicated by overbars instead of minuses, so that, for example, the Miller indices 1, 2, -2 would be written $12\bar{2}$.

To get some geometric feeling for Miller indices, you need to know that the plane containing the three points $(1/h)\mathbf{v}_1$, $(1/k)\mathbf{v}_2$, and $(1/l)\mathbf{v}_3$ is a lattice plane with Miller indices hkl . Three examples are shown in Figure 9.4.

¹ More correctly, the length of $\mathbf{N}$ can be adjusted so that h, k, l become relatively prime integers.

FIGURE 9.5 (Left) Standard pyramidal ice crystal as hypothesized by Steinmetz and Weickmann (same as Figure 8.2). The crystal has prism, basal, and $\{10\bar{1}1\}$ pyramid faces. The angle x is 28° [Eq. (9.6) with $c/a = 1.63$ and $h = l = 1$]. (Right) Same crystal but with the faces identified by their Miller indices.

FIGURE 9.6 Like Figure 9.5 but with $\{10\bar{1}2\}$ faces instead of $\{10\bar{1}1\}$ faces. The angle x is now 46.7° .

FIGURE 9.7 Like Figure 9.5 but with the addition of $\{11\bar{2}1\}$ faces, the simplest second order pyramid faces.

FIGURE 9.8 Crystal easily mistaken for that in Figure 9.5. The crystal has basal faces, $\{11\bar{2}0\}$ prism faces, and $\{11\bar{2}2\}$ pyramid faces. Angle x is 31.5° . In all figures on this page the crystallographic axes are oriented the same; here each a -axis is perpendicular to a (second order) prism face.

For minerals like ice that have hexagonal symmetry there are good geometrical reasons for replacing the triple of Miller indices hkl with the 4-tuple $hkil$, where $i = -(h+k)$. We will usually do so, as in Figure 9.4. This corresponds to introducing another axis, the a_3 -axis, as shown in the figure. The a_1 , a_2 , and c -axes are in the directions of $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, while the a_3 -axis is in the direction of $-(\mathbf{v}_1 + \mathbf{v}_2)$.

Each crystal face is part of what is known as a crystallographic form. The form is the collection of faces related to the given face by the internal symmetries of the crystal.² Curly brackets distinguish the form from the face. That is, $\{hki\}$ is the form containing the face $hkil$. In Figure 9.5 the $\{0001\}$ form consists of the two basal faces, and the $\{10\bar{1}0\}$ form consists of the six prism faces. The $\{10\bar{1}1\}$ form consists of the twelve inclined faces. Technically the $\{10\bar{1}1\}$ form is a hexagonal dipyrmaid, but we nevertheless refer to its faces as pyramid faces. (The crystal at the right in Figure 7.1 consists of a single dipyrmaid.)

The faces $\{h0\bar{h}l\}$ and $\{hh\bar{2}hl\}$ are known as first order and second order faces, respectively. They are prism faces if $l = 0$ and they are (di-)pyramid faces otherwise. The crystals in Figures 9.5 and 9.6 have first order prism and pyramid faces, and the crystal in Figure 9.8 has second order prism and pyramid faces. The crystal in Figure 9.7 has first order prism faces, and it has both first and second order pyramid faces. Perhaps from Figures 9.9 and 9.10 you can see the somewhat subtle difference between first and second order faces. It is a difference that has plagued halo theory historically.

There has been a lot to absorb here. The main point is that crystallography gives us an idea of what crystal faces to expect—they are the lattice planes with small Miller indices.

What, then, can we expect for the faces on a real ice crystal? That is, what might their Miller indices be? All else being equal, we would expect the most common faces to be the $\{0001\}$, $\{10\bar{1}0\}$, and $\{10\bar{1}1\}$ faces. The $\{0001\}$ and $\{10\bar{1}0\}$ faces are of course the familiar basal and prism faces, and they are indeed common. The $\{10\bar{1}1\}$ faces have to be our tentative first choice for the inclined faces. Other top candidates for the inclined faces would be the $\{10\bar{1}2\}$ and $\{11\bar{2}1\}$ faces. Again see Figures 9.5–9.7.

We are now close to being able to guess the wedge angles on a real pyramidal ice crystal.

² For ice there are 24 such symmetries. With the c -axis vertical, there are six rotations about the vertical axis, six reflections in vertical planes, and then each of the preceding twelve operations followed by reflection in the horizontal plane. The group of these 24 symmetries is often denoted by its Hermann–Mauguin symbol $6/m \ 2/m \ 2/m$.

Angle x from Miller indices *The inclination angle x between the $\{hkl\}$ faces ($h+k+i=0$) and the crystal axis is given by*

$$\tan x = \frac{\sqrt{3}}{2} \frac{a}{c} \left| \frac{l}{h} \right| \quad \text{if } k = 0 \quad (\text{first order faces}) \quad (9.6)$$

$$\tan x = \frac{1}{2} \frac{a}{c} \left| \frac{l}{h} \right| \quad \text{if } h = k \quad (\text{second order faces}) \quad (9.7)$$

See Figures 9.9 and 9.10.

Bravais, who was one of the pioneers of crystallography, knew all of this a century and a half ago. That is, he was aware of something like our Table 9.1 and Eqs. (9.6) and (9.7). But to calculate the wedge angles on a pyramidal ice crystal, he still needed to know x , or he needed to know c/a together with the Miller indices of the pyramid faces. Not knowing c/a , he used a value of x inferred from Clarke's observation of rhombohedral crystals (Chapters 7 and 11). We now think this was a mistake. Much later, Steinmetz and Weickmann calculated the wedge angles differently. Using Barnes' $c/a = 1.63$ and taking $h = l = 1$ in Eq. (9.6), they calculated $x = 28^\circ$. In conjunction with Table 9.1, this then gives the wedge angle values listed in Table 8.1. This is our approach as well. In short, the wedge angles in Table 8.1 are calculated from the c/a ratio of ice by assuming that the pyramid faces are the $\{10\bar{1}1\}$ faces.

What about some of the other crystallographically likely faces? We will see in the next chapter that the Steinmetz–Weickmann value $x = 28^\circ$ looks about right for most real ice crystals. The $\{10\bar{1}2\}$ faces and the $\{11\bar{2}1\}$ faces, mentioned above as being candidates for the inclined faces, have $x = 46.7^\circ$ and $x = 17.1^\circ$, respectively, and neither gives a crystal shape resembling the shape of real ice crystals. In fact, as can be seen from Table E.2 of Appendix E, the only plausible alternatives to the $\{10\bar{1}1\}$ faces of Steinmetz and Weickmann are the $\{11\bar{2}2\}$ faces, for which $x = 31.5^\circ$. These Miller indices are small enough to deserve consideration, and the resulting crystal shape (Figure 9.8) and halos are not so different from those for the $\{10\bar{1}1\}$ faces. Careful observations, however, will distinguish the two possibilities, and the observations favor the $\{10\bar{1}1\}$ faces, especially if one looks at the halos arising in preferentially oriented crystals.

So we believe that Steinmetz and Weickmann were right and that the pyramid faces on ice crystals are normally the $\{10\bar{1}1\}$ faces. If so, then the situation is complicated enough. But consider what it might have been. There seems to be no a priori reason why ice crystals should not exhibit other faces. Conceivably we could have had ice crystals like the one in Figure 9.7, for example, which has $\{10\bar{1}1\}$ and $\{11\bar{2}1\}$ pyramid faces in addition to the usual prism and basal

$$\begin{aligned}
 \tan x &= \tan \angle OCB \\
 &= \frac{|\mathbf{OB}|}{|\mathbf{OC}|} \\
 &= \frac{\sqrt{3}}{2} \frac{|\mathbf{OA}|}{|\mathbf{OC}|} \\
 &= \frac{\sqrt{3}}{2} \frac{a}{c} \frac{l}{h}
 \end{aligned}$$

FIGURE 9.9 Derivation of Eq. (9.6), which gives the inclination angle x for first order faces. The grey triangle is the $h0\bar{h}l$ face; it is parallel to the a_2 -axis and intercepts the a_1 , a_3 , and c -axes at a/h , $a/(-h)$, and c/l , the denominators of the fractions being given by the Miller indices as usual. Thus $|\mathbf{OA}| = a/h$ and $|\mathbf{OC}| = c/l$. Since $\angle OAB = 60^\circ$, the derivation proceeds as at the right.

$$\begin{aligned}
 \tan x &= \tan \angle OCB \\
 &= \frac{|\mathbf{OB}|}{|\mathbf{OC}|} \\
 &= \frac{1}{2} \frac{|\mathbf{OA}|}{|\mathbf{OC}|} \\
 &= \frac{1}{2} \frac{a}{c} \frac{l}{h}
 \end{aligned}$$

FIGURE 9.10 Derivation of Eq. (9.7), which gives x for second order faces. The grey triangle is the $hh\bar{2}h̄l$ face; it intercepts the a_1 , a_2 , a_3 , and c -axes at a/h , a/h , $a/(-2h)$, and c/l . Thus $|\mathbf{OA}| = a/h$ and $|\mathbf{OC}| = c/l$, the same as in Figure 9.9, but now $\angle OAB = 30^\circ$.

faces. Most halo theorists have never worried about such crystals, and indeed no such ice crystals have been recognized unequivocally, at least not among crystals formed in the open atmosphere. But some depth hoar crystals, which grow within the snowpack, have been reported to have highly exotic faces [16]. And there are other minerals, such as beryl, which have hexagonal symmetry similar to that of ice, and which sometimes come in complicated crystals. It could have been the same for the ice crystals that make halos.

Incidentally, if it *had* been the same, you could still have computed the wedge angles on the crystals, so long as you knew the Miller indices of the faces. [Use Eqs. (9.1), (9.5), and (E.11).] And once you have the wedge angles, you have the halo radii.

Before moving on, we invite you to look again at the figures from this chapter, if you are not yet comfortable with Miller indices. In all of Figures 9.4–9.8 the crystallographic axes are oriented identically and the c/a ratios are the same, so that faces with the same Miller indices are oriented identically when they appear in different figures. Thus the $\bar{1}101$ face, which is face 15 in Figures 9.5 and 9.7, looks the same both in those figures and in Figure 9.4. The $01\bar{1}2$ face, which is face 14 in Figure 9.6, looks the same there and in Figure 9.4.

Pyramidal Ice Crystals

In Chapter 8 we saw how to calculate halo radii from wedge angles. Then in Chapter 9 we saw how to calculate the wedge angles using crystallographic principles, much as Steinmetz and Weickmann had done. None of this would be convincing if the calculated halo radii were not consistent with the observed radii of real halos or if the calculated wedge angles were not consistent with the observed wedge angles of real ice crystals. And indeed not everyone was convinced by the results of Steinmetz and Weickmann. At that time, however, there were no photographs of odd radius halos and only a few photographs of pyramidal ice crystals. In Chapter 8, with the benefit of recent photographs, we examined several halo displays and found encouraging agreement between the predicted radii of the halos and the actual radii. Now we will look at some real pyramidal ice crystals. We will see that the real crystals do indeed look like the hypothetical crystal of Steinmetz and Weickmann.

In fact the resemblance is clear at a glance. Compare Figure 9.5, showing the hypothetical crystal, with almost any of the pyramidal crystals shown in the present chapter. Remember that the sizes of the crystal faces relative to one another are immaterial, so in Figure 10.4, for example, it is all right that one of the basal faces on the pyramidal crystal is small, and it is all right that half the pyramid faces are missing. What matters is the orientations of the faces relative to one another, and the orientations do indeed look right.

To get a more quantitative feeling for the resemblance between the hypothetical and the real, we need to introduce another angle, denoted by x' . Recall that angle x is the angle between the crystal axis and an inclined (i.e., pyramid) face. Angle x' is the angle between the crystal axis and an inclined *edge*. In the crystal photos it is normally x' , rather than x , that can be measured. That is, no

pyramid face is apt to be seen edge-on, as would be necessary for measuring x . But if the crystal is lying on one of its prism faces, then the crystal axis and an inclined edge will both be parallel to the plane of the photo, so that x' can be measured. Angle x' is as good as angle x , in the sense that each determines all wedge angles of the crystal.

For our hypothetical crystal, with $x = 28^\circ$, the theoretical value of x' turns out to be 31.5° . We have marked angles of 31.5° in the crystal photos in Figure 10.6. We stress that the 31.5° angle is the angle between the line segments in the photos, and that the line segments are not necessarily exactly parallel to the relevant crystal edges. However, in most cases we think the parallelism is nearly perfect and that therefore x' is indeed very nearly 31.5° .

So the agreement between the hypothetical crystal and the real crystals is quite good. You may be suspicious that the crystal photos came first, and that we then concocted the hypothetical crystal, but historically it was the reverse, or nearly so.

The pyramidal crystals of this chapter should be contrasted with the bullet crystals of Figure 2.7. The tapered surfaces of bullets are not true pyramidal faces and in fact are not even planar. Pyramidal crystals make odd radius halos, bullet crystals do not.

Large pyramidal crystals often come with some very strange looking company. Many of the crystals in that company nearly defy description, even in a good photograph, but there are others that we can make at least limited sense of. Figure 10.2 illustrates one type. There the left-hand crystal, a pyramid crystal, has shallow depressions in some of its faces, so that the faces are not perfectly planar. This gives a clue as to what is going on with the right-hand crystal. It, too, is a pyramid crystal, but the depressions now are veritable pits, nearly destroying the identity of the crystal. Nevertheless, the pits probably show where the pyramid faces would have been, had the crystal been better formed. Another example of this sort of pitted pyramid is the upper left crystal in Figure 10.12. Such crystals look at first like old crystals, badly deteriorated, perhaps blown up from the snow surface, but our experience is that they are not. Incidentally, we do not mean to imply that the pits have been somehow excavated in the faces. Rather the crystal growth in the centers of the faces simply cannot keep up with the growth along the edges.

It is clear from photos like Figure 10.2 that not all ice crystals have perfectly planar faces, and that the Miller index crystallography of Chapter 9 sometimes fails to describe real crystals. The failure can be only slight, as for the left-hand crystal, or it can be dramatic, as for the right. Of course you needn't go to pyramid crystals to see this failure. Ordinary column crystals often have indentations in their basal faces, and although in some crystals these indentations conform to

Miller index crystallography, as in the left center photo of Figure 2.6, where the indentation surfaces consist of stepped prism and basal faces, in other crystals the indentations are curved, as in the left center photo of Figure 2.5. Even the familiar stellar crystals often have curved basal “faces.” This is obvious for the lower right crystal in Figure 2.1 and more subtle in the crystals of Figure 2.8. In the latter figure the basal face of the left-hand crystal is truly planar in the center, but then it tapers towards the edges, leaving tiny or non-existent prism faces. The basal faces of the right-hand crystal are truly planar nowhere.

Of the crystals shown in this chapter, those in the lower left photo of Figure 10.6 and in Figures 10.13–10.16 are from Fairbanks, Alaska, and the rest are from the South Pole. In many cases the Fairbanks crystals formed and grew in moisture from power plant plumes. Because of these special conditions, it is conceivable that the crystals themselves are somewhat special and not representative of crystals grown at normal cloud levels in the atmosphere. We do not think so—we are guessing that the pyramidal and more exotic crystals are due mainly to the low temperatures rather than, say, to contaminants in the plumes. But we may be wrong, and the possibility needs to be entertained that these crystals are special. If so, then the halos that they make might also be special.¹

¹ Low level halo displays in Fairbanks appear in Figures 8.3, 8.4, 8.5, 8.14, 8.16, 15.2, 15.3, 15.4, 15.10, 15.11, 15.12, 15.13, 15.17, 15.18, 15.20, 18.8.

FIGURE 10.1 Pyramid crystals.

FIGURE 10.2 Pyramid crystals.

FIGURE 10.3 Tiny pyramid faces.

FIGURE 10.4 Pyramid crystal above.

FIGURE 10.5 Pyramid crystal below.

FIGURE 10.6 Pyramid crystals. Line segments have been placed as nearly parallel as possible to crystal edges, but with the angle between line segments fixed at 31.5°. The value 31.5° is the theoretical value for the angle χ' between appropriate *edges* on a crystal whose pyramidal faces are the $\{10\bar{1}1\}$ faces. The crystals are from 100 to 180 microns in length.

FIGURE 10.7 Pyramid crystal at left.

FIGURE 10.8 Complex crystals often associated with pyramid crystals.

FIGURE 10.9 Twinned crystal? Such crystals often occur with pyramid crystals.

FIGURE 10.10 Twinned crystal at lower left?

FIGURE 10.11 Two pyramid crystals among more exotic crystals.

FIGURE 10.12 Well formed pyramid crystal among more exotic crystals.

FIGURE 10.13 ESEM photo of atmospheric ice crystals collected in Fairbanks. Probably only the larger crystals here are big enough to make halos. The wispy object at upper right is extraneous. The 100μ label refers to the entire scale bar—black and white together. Photo © Kenneth P. Severin.

FIGURE 10.14 Crystals from the same crystal sample as in Figure 10.13. In the upper photo the black and white scale bar is 100μ , and in the lower photos, 50μ . The evident crystal damage is probably due to the electron beam. Photos © Kenneth P. Severin.

FIGURE 10.15 Pyramid crystals, both from the same crystal sample. At the right the view is looking directly down the c -axis. The black and white scale bar is 25μ . Photos © Kenneth P. Severin.

FIGURE 10.16 Crossed pyramidal plate crystal at left. Our experience is that most crossed plate crystals, when seen with the ESEM, turn out to be crossed pyramidal plates. The crystal with the pyramidal faces at the right is not a common type and will go unnamed. The black and white scale bar is 35μ at left, 150μ at right. Photos © Kenneth P. Severin.

The Search For Angle x

We have tried to show that pyramidal ice crystals are real and that their pyramid faces are normally the simplest possible, namely, the $\{10\bar{1}1\}$ faces. Theoretically these faces are expected to make an angle of 28° with the c -axis of the crystal. Photographs of ice crystals show pyramid faces that appear to be the $\{10\bar{1}1\}$ faces, and photographs of odd radius halos generally show halos that look like those theoretically expected from crystals with $\{10\bar{1}1\}$ faces. So we think we are making some headway in our effort to understand pyramidal ice crystals and odd radius halos. Getting this far, however, occupied the better part of two centuries, and it was not a smooth ride. We find this struggle for understanding to be fascinating in its own right, but we also think there are lessons to be learned from it. In this chapter we therefore look at it in some detail. It is a matter of fleshing out the history outlined in Chapter 7.

Ice has long been thought, correctly, to have hexagonal, or perhaps trigonal, symmetry. Johannes Kepler, for example, in 1611 in *The Six-Cornered Snowflake* [35] remarked on the hexagonal symmetry of ice and speculated on the internal structure that might be responsible for it. Because of the hexagonal or trigonal symmetry, people could reasonably conjecture the existence of pyramidal or rhombohedral crystals of ice, even if nobody had ever seen one. Although other crystal shapes are conceivable, a pyramidal crystal became the standard crystal model used by people studying odd radius halos. When complete, the crystal would have two basal faces, six prism faces, and twelve pyramid faces, as in, say, Figure 9.5. But the value of angle x —the inclination of the pyramid faces to the c -axis—does not follow from the symmetry, and, as we have seen, it is crucial

for halo theory, since it determines the wedge angles on the crystal and hence the radii of any resulting circular halos. So a major part of the effort to understand odd radius halos was directed at finding the most plausible value for x . This was not so easy, given the meager record of pyramidal crystal observations and the confused record of odd radius halo observations.

Before Bravais

We mentioned in Chapter 7 that Mariotte in the seventeenth century had briefly entertained pyramidal crystals as a possible explanation for the 22° halo. Each of his crystals was a triangular pyramid in the strict sense: it had a triangular basal face and three pyramid faces, nothing more. Mariotte took $x = 30^\circ$.

Venturi [83], in the early nineteenth century, also used pyramidal crystals to explain various halos, most notably the tangent arc. (He was not treating odd radius halos.) In fact, all of Venturi's columnar crystals had pyramidal terminations, with no basal faces. Venturi took $x = 60^\circ$.

Most of Venturi's explanations of individual halos did not pan out, and reading Venturi's book makes one appreciate how difficult the subject of halos was for its pioneers. We mention Venturi here mainly to show that pyramidal crystals, if only hypothetical, were indeed part of the halo conversation at this time.

Galle [18] in 1840 used pyramidal crystals to explain the 22° halo, though not in the same way as Mariotte. Galle had made many careful measurements

FIGURE 11.1 Pyramidal crystal of Galle, with $x = 60.32^\circ$. Compare with Figures 9.5 or 9.6.

of halos and had concluded that the average measured radius of the 22° halo was slightly less than what he calculated theoretically for a wedge angle of 60° . He reasoned that the 22° halo was actually a mix of two overlapping halos, one made by the usual 60° wedge, and one made by a slightly smaller wedge and thus having a slightly smaller radius. He calculated that a pyramidal crystal with inclination $x = 60.32^\circ$ would give the desired smaller

radius. In Figure 11.1 it is wedge 13 23, or 14 24, etc., that has wedge angle $\alpha = 180 - 2x = 59.35^\circ$, which was the angle that Galle wanted.

If you use our Table 9.1 to calculate all of the wedge angles on Galle's pyramidal crystal, you will find three wedge angles all very close to 60° . Each would give a halo with radius close to 22° , so for Galle the 22° halo was actually a mix of three overlapping halos.

But more to the point: When you compute the wedge angles, you will also find

FIGURE 11.2 Pyramidal crystal and two rhombohedral crystals chosen to illustrate the close relation between the (di-)pyramidal and rhombohedral forms. The dark faces on the pyramidal crystal correspond to the faces on the dark rhombohedron, the light faces correspond to the faces on the light rhombohedron. The two rhombohedra differ from each other only by a 60° rotation about the c -axis (vertical). The rhombohedra shown here are shaped like those reported in 1821 by Clarke, who measured the interfacial angles and found them to be 60° and 120° . From these values Bravais calculated the inclination x between any face and the c -axis to be 54.7° .

angles that theoretically give rise to halos having radii of about 9° , 18° , and 31° . Galle noted that there were a number of reports of halos having radii more or less agreeing with these three theoretical radii. He also noted, however, that there were other reported radii as well, radii that were not explained by his hypothetical crystal, and he made no claim to have explained odd radius halos.

Galle's x -value did not catch on. Instead it was a value proposed by Bravais, in 1847, that came to be accepted and then long retained. The appeal of Bravais' x -value was that it was supposed to be based on a real crystal observation. And, in Bravais' hands, it led to passable explanations for many odd radius halos. To explain Bravais' derivation of x , we first need to say more about the symmetry of ice.

The symmetry of ice

Ice, like the mineral beryl, for example, turns out to have full hexagonal symmetry, technically described by the Hermann–Mauguin notation $6/m\ 2/m\ 2/m$. The simplest non-prismatic crystal having $6/m\ 2/m\ 2/m$ symmetry should be a hexagonal dipyramid (Figure 11.2, left). For a long time, however, it seemed possible that ice had a lower symmetry, perhaps more like quartz, say, which has symmetry technically described by the Hermann–Mauguin notation 32 . In that case the simplest non-prismatic crystal would be a rhombohedron (Figure 11.2, middle or right), or perhaps a trigonal dipyramid.

Though not necessary for what follows, you can get an inkling of what is meant by low symmetry by taking the lattice of Figure 9.3 and replacing each lattice point by a little upright equilateral triangular prism, with all of the prisms having the same orientation. The resulting configuration of prisms has some of

the symmetry of the lattice, but not all; it has “lower” symmetry. For example, you can rotate the original lattice 180° about the c -axis, and the lattice appears unchanged, but you cannot do the same for the configuration of prisms.

The question of the correct symmetry for ice goes well beyond its marginal relevance to halo theory, and a considerable effort during the twentieth century was made to answer it. The difficulty in determining the symmetry is that, first, the external form of ice crystals gives nothing away. Or rather, their external form can at times even argue *against* $6/m\ 2/m\ 2/m$ symmetry; examples would be the triangular ice crystals in the left-hand photo of Figure 2.2, or the blatantly asymmetric crystal at the upper right in Figure 10.4.¹ Secondly, the hydrogen atoms in ice are nearly invisible to X-rays, so that X-ray diffraction does not give a complete picture of the internal structure of ice. But in spite of these and other difficulties, a consensus eventually emerged in favor of $6/m\ 2/m\ 2/m$ symmetry. Much more about the symmetry of ice can be found in a recent book by Petrenko and Whitworth [58]. Older but still very thoughtful discussions can be found in articles by Dobrowolski [15], Barnes [3], and Steinmetz [71].

Bravais

For the purposes of halo theory, the question of the true symmetry of ice is not critical. In Figure 11.2 it makes no difference whether the upper left crystal is considered to be a hexagonal dipyrmaid with $6/m\ 2/m\ 2/m$ symmetry or a combination of two rhombohedra with $3\ 2$ symmetry. So when Clarke (Chapter 7) saw rhombohedral crystals and apparently succeeded in measuring their interfacial angles, it appeared to be the key to finding angle x and therefore the likely shapes of any pyramidal or rhombohedral crystals. Clarke had found the angle between faces 13 and 15 to be 120° , and from it Bravais computed $x = \tan^{-1}\sqrt{2} = 54.7^\circ$, essentially using the 13 15 line of our Table 9.1. The crystals then would be shaped like those in Figures 11.2 or 11.3. If, as might reasonably be guessed, Clarke was seeing the $\{10\bar{1}1\}$ faces, then the Bravais value for c/a would be $\sqrt{3/8} = 0.612$, from Eq. (9.6). But c/a was not crucial at this stage, as we will see.

This value $x = 54.7^\circ$ pervaded the theory of odd radius halos for over a century. It seems to have originated with Bravais and then been picked up by others, such as Pernter and Exner [57] in the early twentieth century and Visser [84] in the mid-twentieth century. For Bravais in 1847 the choice of 54.7° was reasonable, based as it was on Clarke’s observation, which had been made only a couple of decades earlier. (Rhombohedral ice crystals had also been reported by David Brewster [10],

¹ One can argue that the crystal in Figure 10.4 might have fallen with its c -axis vertical, in which case the two ends of the crystal would have been exposed to different conditions for growth, thus explaining the asymmetry. But it is much harder to reconcile the triangular crystals of Figure 2.2 with $6/m\ 2/m\ 2/m$ symmetry.

FIGURE 11.3 Pyramidal crystal inferred from Clarke’s rhombohedral crystals by Bravais. The crystal is like the one at the left in Figure 11.2 but with prism and basal faces in addition to the pyramid faces. Angle x is 54.7° . Several generations of halo theorists regarded crystals like these—either the one here or those in Figure 11.2—as plausible models for real ice crystals, but no crystals with $x = 54.7^\circ$ have been reported since Clarke.

TABLE 11.1 Wedge angles and halo radii for the Bravais-Clarke crystal of Figure 11.3.

Wedge	Wedge angle α	Halo radius $\Delta_{\min}$
13 25	33.6°	10.9
13 2	35.3	11.5
13 6	54.7	19.3
3 5	60.0	21.8
13 23	70.5	27.8
13 5	73.2	29.5
1 3	90.0	45.7

in 1834, but Brewster had not been able to measure the interfacial angles.) But as years passed and nobody was able to duplicate Clarke’s measurement, the choice $x = 54.7^\circ$ should have drawn more skepticism than it did. Actually, it did draw some, eventually, from A. B. Dobrowolski, who in his *Les cristaux de glace* [15] in 1916 dismissed Clarke’s observation and stressed that the c/a ratio of ice was still unknown. But Dobrowolski’s caution did not deter subsequent halo theorists, who continued to take Clarke’s observation at face value.

At any rate, Bravais in 1847 had little else to go on. Taking $x = 54.7^\circ$, he computed the wedge angles on the resulting pyramidal crystal, essentially evaluating the last column in our Table 9.1. The results are given in Table 11.1, along with the corresponding halo radii.

Bravais treated seven circular halos. Their measured radii were 14.2° (Heiden’s halo), 20° (Burney’s halo), 25° – 28° (Scheiner’s halo), 36° (Feuillée’s halo), 90° (Hevel’s halo), and of course 22° and 46° as usual. As explained in connection with Eq. (8.2), the radius of 90° is far too large to be explained by a ray path that simply enters one crystal face and then directly exits another, as we are assuming, and so Hevel’s halo is a separate problem. As can be seen from Table 11.1, the radii 20° , 22° , and 46° are close to halo radii predicted from the Bravais-Clarke pyramidal crystal. That still left Bravais with radii of 14.2° , 25° – 28° , and 36° to account for. He resorted to a new crystal for each.

What would a reasonable crystal be? Probably it would be another pyramidal crystal, but with x to be determined. Some candidates for x are given by the following principle, which we will call the Rational Tangents Principle.

Rational Tangents Principle *If $\tan x / \tan x_0 = v/u$ for some small positive integers u and v , and if x_0 is a crystallographically likely inclination angle, then so is x . (See Appendix E.)*

Taking $x_0 = 54.7^\circ$ from the Clarke rhombohedron, Bravais must have tried various fractions v/u until he came up with the desired halo radii. For Heiden's halo he chose $v/u = 2/3$, which gives $x = 43.3^\circ$. The wedge angles and corresponding halo radii for the resulting pyramidal crystal can be calculated from our Table 9.1 and Eq. (8.1); they are listed in Table 11.2. Two of the resulting halo radii—the ones for the wedges 13 25 and 13 6—are indeed close to 14.2° , the value measured by Heiden. Similarly, Bravais chose $v/u = 2/1$ to produce Scheiner's halo, and $v/u = 4/1$ to produce Feuillée's halo. In all, Bravais postulated four essentially different pyramidal crystals.

In the Rational Tangents Principle it turns out that if the $\{hkl\}$ faces are the faces with inclination x_0 , then the $\{uh\ uk\ ui\ vl\}$ faces are the faces with inclination x . If we assume that the pyramid faces on the Bravais-Clarke crystal are the $\{10\bar{1}1\}$ faces, then the pyramid faces on the other three crystals are the $\{30\bar{3}2\}$ faces ($u = 3, v = 2$), the $\{10\bar{1}2\}$ faces, and the $\{10\bar{1}4\}$ faces.

Things were getting complicated. To explain Heiden's halo, for example, Bravais not only had to come up with the $\{30\bar{3}2\}$ faces. Once he had done so and essentially constructed our Table 11.2, he then had to explain why some of the predicted halos in the table did not seem to occur. (In fact, here Bravais seems to have been using a crystal with only faces 13, 15, 17, 23, 25, 27; this eliminated all but the 14.3° and 51.4° halos from Table 11.2.) He also had to explain why there were so many potential crystal faces that were theoretically more likely than the $\{30\bar{3}2\}$ faces (Table E.3 of Appendix E) but which did not seem to appear in reality. (He didn't.) Analogous excuses had to be made for the $\{10\bar{1}2\}$ and $\{10\bar{1}4\}$ faces that he had invoked.

The party line

Neither the c/a ratio nor Miller indices appear explicitly in the Rational Tangents Principle. Thus in finding angle x for the crystal that was supposed to make Heiden's halo, Bravais did not need to know c/a and he did not need to assume that the pyramid faces of the Bravais-Clarke crystal were the $\{10\bar{1}1\}$ faces. Renaming these faces to be, say, the $\{20\bar{2}1\}$ faces would change the indices of the other pyramid faces and it would change c/a , but it would not change the

TABLE 11.2 Wedge angles and halo radii for the pyramidal crystal used by Bravais to explain Heiden’s halo. Angle x for this crystal is 43.3° .

Wedge	Wedge angle α	Halo radius $\Delta_{\min}$
13 25	42.7°	14.3°
13 6	43.3	14.5
13 2	46.7	15.9
3 5	60.0	21.8
13 5	68.7	26.6
13 24	78.1	33.2
13 16	86.6	41.3
1 3	90.0	45.7
13 23	93.4	51.4

inclination angles x_0 and x , and it would not change the crystal shapes themselves. The pyramid faces of the Bravais-Clarke crystal could even be renamed to be second order faces—the $\{11\bar{2}1\}$ faces, for example—so long as the prism faces were renamed as well.

If you try to read the literature on odd radius halos, you will find that the Rational Tangents Principle was a sort of party line, the fundamental tool for incorporating the crystallography. It allows you to generate plausible inclination angles x from a known inclination angle x_0 even if you do not know c/a and even if you do not know the Miller indices. However, the principle does not generate all such angles x . If x_0 is associated with a first order face then so is x , and if x_0 is associated with a second order face then, again, so is x . The principle also has the disadvantage that, unless it is applied conservatively, that is, unless the integers u and v are taken to be very small indeed, you quickly end up with faces that are highly unlikely.

Besson

After Bravais in 1847 the next significant attacks on the problem of determining angle x did not come until the early twentieth century, when Louis Besson and W. J. Humphreys each took a new look at the problem. They ended up nearly at each other’s throats.

Besson’s [5–7] idea was to select the halos whose radii had been most reliably measured and then to choose angle x so as to best fit the measured radii. Besson constructed a set of curves like those in Figure 11.4, where for each wedge there is a curve that gives the halo radius as a function of x . That is, he essentially

FIGURE 11.4 Besson's method of inferring x from measured halo radii. As was usual, the crystal was assumed to consist of six prism faces, two basal faces, and a hexagonal dipyrmaid determined by x . Each curve in the diagram gives the theoretical halo radius $\Delta_{\min}$ as a function of x for a particular wedge of the crystal. The three lowermost horizontal dashed lines correspond to the measured radii of van Buijsen's, Rankin's, and Burney's halo at the time of Besson. Besson looked for x -values for which three of the theoretical halo radii would coincide with the three measured radii. The value $x = 25^\circ$ was best, but $x = 28^\circ$ was nearly as good, as can be seen from the figure. The two crystals shown are for $x = 10^\circ$ and $x = 80^\circ$ and are included just to give a feeling for the range of crystal shapes under consideration.

used Table 9.1 and Eq. (8.1) to compute the wedge angles and then the halo radii $\Delta_{\min}$ as functions of x . Once he had constructed all of the curves giving the halo radii, he then looked for the value of x that gave halo radii closest to his measured radii. The halos that he selected to analyze were the halos of van Buijsen, Rankin, and Burney, which had measured radii of 8.5° , 17.5° , and 19° —presumably our 9° , 18° , and 20° halos.

Besson found two candidates for x , namely, angles near 25° and angles near 28° . Then, as Besson put it,

The inclination in the vicinity of 25° is in better accord with the observations of halos and it is also in harmony with the value $54^\circ 44'$ which Bravais has given of a crystallographic observation of Clarke, which is $3 \tan 25^\circ 14.4' = \tan 54^\circ 44.1'$, and this has led me finally to maintain as particularly probable, the value $25^\circ 14.4'$. (Besson [7, page 254])

In other words, Besson was using the Rational Tangents Principle with the Bravais-Clarke value $x_0 = 54.7^\circ$ and with $\tan x / \tan x_0 = v/u = 1/3$. The pyramid faces on the Besson crystal would therefore have been the $\{30\bar{3}1\}$ faces. Or you can take the point of view that Besson's pyramid faces are the $\{10\bar{1}1\}$ faces and that the faces on Clarke's rhombs are the $\{10\bar{1}3\}$ faces.

As can be seen from Figure 11.4, Besson got good agreement between his predicted halo radii and the measured radii that he had chosen to analyze—the supposedly most reliable. The halo that might have tipped the scales away from $x = 25^\circ$ and toward $x = 28^\circ$ is the halo of Feuillée—presumably our 35° halo²—which Besson did consider, but as something of an afterthought. Besson knew of four observations of Feuillée's halo, the reported radii being about 32° , 33° , 35° , and 36° . He took the 32° measurement to be most reliable, and this favored $x = 25^\circ$ over $x = 28^\circ$.

Our Figure 8.3 would have been decisive in the other direction. For Feuillée's halo the value $x = 28^\circ$ gives a theoretical radius $\Delta_{\min} = 34.9^\circ$ (Table 8.1), whereas $x = 25^\circ$ gives $\Delta_{\min} = 31.9^\circ$.³ The red dots in Figure 8.3, at 34.9° from the sun, appear to be right at the inner edge of the halo, whereas the yellow dot, 32° from the sun, is nowhere close. Had Besson had the luxury of access to this figure, he surely would have gone with 35° rather than 32° for the radius of Feuillée's halo, and he would have concluded $x = 28^\circ$. In general, Besson perhaps got a bit lucky regarding his choice of halos to analyze, but his luck ran out—we think—when he came to Feuillée's halo. We should add, however, that even today we have very few good measurements of the radius of this halo.

² The halo seen by Feuillée [17] himself was almost certainly *not* our 35° halo, and it is an accident that his name has been attached to the 35° halo. The halo display that Feuillée reported is a mystery to us.

³ Use the 13 15 line of Table 9.1 together with Eq. (8.1).

Humphreys

D. M. Dennison [13], using X-rays even before Barnes, had found $c/a = 1.62$. Beginning about 1922, Humphreys [29–31] took Dennison’s c/a -value and tried to derive x :

...by X-ray analysis it has been shown that... the axial ratio (longitudinal to lateral) is almost exactly 1.62. Clearly, then, from the laws of crystallography, the ratio of the height of the pyramidal end of an ice crystal to the inner radius of its base (a lateral axis), must also be 1.62, or some multiple thereof, expressible in either a small whole number or a fraction whose numerator and denominator both are small whole numbers. If, now, we multiply 1.62 by $4/3$, a factor entirely allowable, we obtain a pyramid whose sides are inclined $24^\circ 51'$ to the longitudinal axis. (Humphreys [31, page 528])

From angle x he then calculated the wedge angles and the corresponding halo radii. His resulting radii were of course close to those of Besson, since the Humphreys and Besson x -values were themselves close.

We think Humphreys made a mistake. The inner radius that he talks about in the quote is not a crystallographic a -axis, at least not if he is referring to first order pyramids. For first order pyramids the ratio of height to inner radius is, in our notation, $\cot x$, which is, from Figure 9.9,

$$\frac{2}{\sqrt{3}} \frac{h}{l} \frac{c}{a} \quad (\text{first order faces})$$

But $2h/(\sqrt{3}l)$ is not a quotient of small whole numbers. For second order pyramids the ratio of height to inner radius is, from Figure 9.10,

$$\frac{2h}{l} \frac{c}{a} \quad (\text{second order faces})$$

The ratio would therefore be c/a if the pyramid faces were the $\{11\bar{2}2\}$ faces (i.e., $h=1$ and $l=2$). And the ratio would be $(4/3)c/a$, giving $x = 24.8^\circ$ as Humphreys claimed, if the pyramid faces were the $\{22\bar{4}3\}$ faces.

We are guessing that Humphreys thought he was talking about the $\{40\bar{4}3\}$ faces when he found $x = 24.8^\circ$, but in one sense it does not matter much, because neither the $\{22\bar{4}3\}$ faces nor the $\{40\bar{4}3\}$ faces are at all likely, contrary to his remark that $4/3$ is an entirely allowable factor. Table E.2 gives an inkling of just how unlikely these faces are. In the table there are 31 essentially different crystal faces (forms) that are more likely than the $\{22\bar{4}3\}$ faces. The $\{40\bar{4}3\}$ faces are even less likely.

Thus Humphreys anticipated the approach of Steinmetz and Weickmann. But although his intention of exploiting the known c/a -value was sound, we think that he did so incorrectly and that there was no basis for his conclusion $x = 24.8^\circ$.

This value $x = 24.8^\circ$ has nevertheless enjoyed a long life in the halo literature, where it remained quite healthy into the 1970's and where even today it is not completely dead.

Humphreys may have been more concerned with Hevel's halo (Chapter 18) than with any of the halos that we have been discussing. He had devised an explanation for Hevel's halo that required x to be close to 25° , and this may have colored his assessment of Feuillée's halo. The measured radius of Feuillée's halo, as we saw in connection with Besson, is a good test for the theory; a radius of 32° is consistent with $x = 25^\circ$, a radius of 35° is not. In his *Monthly Weather Review* article [29] of 1922 Humphreys was appropriately cautious about Feuillée's halo:

The radii of these are, roughly, 8° , 17° , 19° , and, perhaps, 32° . The last of these values is based on various crude estimates ranging from about 28° to 33° , or more.

But in the version in his book, *Physics of the Air* [31], the crucial qualification about the large uncertainty in the Feuillée halo radius was missing. Humphreys took the measured radius to be 32° , and as a result his x -value looked better than it really was.

Besson and Humphreys argued with each other over their two approaches and over the virtues of $x = 24.8^\circ$ versus $x = 25.2^\circ$. Today, of course, we think neither of their x -values was correct. When you read about circular halos whose radii are theoretically supposed to be 8° , 17° , or 32° , you are encountering the legacy of Humphreys and Besson; the value $x = 25^\circ$ gives 8° , 17° , and 32° halos, whereas $x = 28^\circ$ gives 9° , 18° , and 35° halos (Figure 11.4).

Steinmetz and Weickmann

We think Steinmetz and Weickmann got it right, in 1947. As already explained, they took Barnes' value of $c/a = 1.63$, assumed that the pyramid faces were the $\{10\bar{1}1\}$ faces, and then calculated $x = 28^\circ$. The wedge angles and corresponding halo radii are then as in Table 8.1.

Visser

In a long chapter in *Handbuch der Geophysik* in 1961, S. W. Visser [84] gave a solid summary of halo theory to that date. His treatment of odd radius halos, however, only serves to convey the confusion that surrounded them. His treatment was basically an elaboration of that of Bravais, but with more reported halo radii to contend with. Even after paring down the many reported radii to those that were supposedly most reliable, Visser had to resort to at least seven essentially different pyramidal crystal forms, as opposed to the four pyramidal forms of Bravais or the

single pyramidal form of Steinmetz and Weickmann. Although he equivocated, Visser in the end stayed with the Bravais-Clarke convention $x = 54.7^\circ$ (for his simplest pyramidal form). He was aware of the work of Steinmetz and Weickmann but was apparently unconvinced by it.

Tricker

In 1976 E. C. W. Goldie, G. T. Meaden, and R. White [20] analyzed an odd radius display that had occurred on Easter of 1974 in England and Holland. They used the Besson approach, trying to find an x -value that would best match the halo radii that they had measured from photographs of the display. They concluded $x = 28^\circ$, but they gave few details, not even their values for the measured radii.

The same display was discussed by R. A. R. Tricker [79] in 1979. Tricker had rediscovered the approach of Steinmetz and Weickmann and thus found $x = 28^\circ$. Tricker's article served to bring the Steinmetz and Weickmann point of view to the attention of English speaking readers. With the appearance of Tricker's article, the Steinmetz and Weickmann approach was squarely on the table. We will make this a stopping point and move on to the history of crystal observations.

Pyramidal crystal observations

We stress again the distinction between bullet crystals (Figure 2.7) and true pyramidal crystals (Chapter 10). Most people who report seeing pyramidal crystals are seeing bullets. Bullet crystals are common, whereas pyramidal crystals—at least large ones—are relatively rare.

There do exist a handful of old observations that cannot be dismissed as bullet crystals. Clarke and Brewster, as already mentioned, each reported seeing rhombohedral crystals. H. Schlagintweit [64] also reported rhombohedral ice crystals, in 1854, found high on Monte Rosa. Héricart de Thury [25] reported seeing pyramidal ice crystals during a cave exploration in France in 1805. His description is magical, but we have no idea what he was seeing. Then there are intriguing microscope observations of pyramidal crystals by A. E. Nordenskjöld [54] in 1860. Nordenskjöld saw three different pyramidal forms on the same crystal, measured the x -values for each, and came up with two possible values for c/a . To our knowledge nobody since then has for sure seen an atmospheric ice crystal having more than one pyramidal form. We would dismiss Nordenskjöld's observation entirely, except that, first, he sounds like a cautious observer, and, second, his three measured x -values turn out to be consistent with the theoretical x -values for the $\{10\bar{1}1\}$, $\{10\bar{1}2\}$, and $\{30\bar{3}1\}$ faces computed from the modern $c/a = 1.63$. Much later,

in 1922, Steinmetz [70] measured $x = 27.97^\circ$ on a large ice crystal in a cave. His x -value is correct, according to modern thinking, but most of the crystals in the cave were, from the sounds of his description, just ordinary frost cups without true pyramid faces.

Good ice crystal photographs began to appear about the beginning of the twentieth century [4, 22, 53, 88], but the first photos that unequivocally show pyramidal crystals seem to be those of Weickmann [86, 87] in the 1940's. His photos are good enough that the interfacial angles on some of the crystals can be measured. Weickmann did so and found angles roughly in agreement with those theoretically expected for $\{10\bar{1}1\}$ faces (and $c/a = 1.63$). The agreement was less than spectacular, however, and Steinmetz and Weickmann did not make a point of it when they wrote their *Zusammenhänge* article [72], the fundamental article on odd radius halos.

The point, instead, was made by Teisaku Kobayashi and Keiji Higuchi [38], who in the 1950's succeeded in growing pyramidal ice crystals in the laboratory. The crystals were grown under slightly peculiar conditions, which raised the question of whether the crystals were truly representative of pyramidal crystals in the atmosphere, but Kobayashi and Higuchi had nevertheless made a major advance. They measured interfacial angles on many of the crystals and concluded that the pyramid faces were indeed the $\{10\bar{1}1\}$ faces. Their published work, as well as that of Kobayashi [37], contains many fine crystal photographs that can be measured relatively easily. By this time, then, there was evidence from the crystals themselves that the $\{10\bar{1}1\}$ faces might indeed be the norm for pyramid faces.

In 1969 Takeshi Ohtake [55] photographed natural pyramidal crystals in Fairbanks, Alaska. The pyramid faces look qualitatively like the $\{10\bar{1}1\}$ faces, but the crystals do not happen to be oriented so that the interfacial angles can be measured.

We conclude this section with what is probably only a curiosity and not significant. In the 1950's K. Ito [34] photographed ice crystals that on first glance appear to have peculiar faces—they are neither basal faces, first order prism faces, nor $\{10\bar{1}1\}$ pyramid faces. In his photos he identified $\{11\bar{2}0\}$, $\{11\bar{2}1\}$, $\{11\bar{2}2\}$, $\{22\bar{4}1\}$, and even higher index faces. The peculiar faces were nearly always flush against the glass of the collecting dish, and it looks to us like the crystals grew on the glass, or perhaps melted or evaporated where they contacted the glass, so that the peculiar faces are not crystallographic faces at all. Weickmann [86, Plates 45 and 46] has photographs of similar crystals, but Weickmann did not claim to see any exotic crystallographic faces in them. We, too, have photographed such crystals, and we do not think the peculiar faces are true crystallographic faces, but we are not positive.

Odd radius halo photographs

In Chapter 7 we mentioned some of the first published photographs of odd radius halos. Like the pyramidal crystal photos, they came too late to help the early halo theorists.

There is another photo that needs to be mentioned, because it has muddied the waters somewhat. The photo was taken by O. M. Ashford in 1956 and interpreted by R. S. Scorer [65] as showing an 8° circular halo. Because the temperature at the time was near freezing, the inference was drawn that pyramidal crystals form under warm conditions. We ourselves see in the photo only a circular lens artifact, the kind that often results when a wide angle camera lens is aimed at the unblocked sun.

20–20 hindsight

The preceding is a summary of the struggle to determine angle x or, in plainer language, the struggle to figure out what a pyramidal ice crystal might look like. The summary is incomplete, but perhaps it is enough. What are we to make of it?

If you study the old work, you will find many mistakes. There was carelessness and there was self-deception. There was far too much uncritical reliance on previous literature, so that mistakes got passed on from author to author. *Caveat lector.*

Many of the mistakes were just computational and must have been nearly inevitable among the massive hand calculations that were required. They caused no lasting harm. But there were substantive mistakes or misconceptions as well, and a couple of them did have long-term implications for the theory. The uncritical reliance on Clarke's rhombohedral crystal observation and on the inferred value $x = 54.7^\circ$, even when the observation had not been reproduced for over a century, corrupted the theory. So did Humphreys' conclusion $x = 25^\circ$. Perhaps it is bad manners for us to be calling attention to these misconceptions, especially with our unfair advantage of 20–20 hindsight, but we think they have proved sufficiently resilient and damaging that they deserve some exposure.

In any case, these mistakes might have been caught, had there been a decent observational record either of pyramidal crystals or of odd radius halos. But there was neither. Halo theorists ended up trying to explain facts that were not facts at all.

What lessons can we take for ourselves from our reading of halo history? In general, it is easy to make mistakes, and we are undoubtedly making our share. We need to be careful.

But more specifically: Among the small circle of halo enthusiasts there has been a shift in orthodoxy regarding the odd radius halos. The shift has been

away from something like the Visser chaos and toward the cleaner and simpler Steinmetz and Weickmann theory. The shift happened in the last few decades, and it happened without a lot of fuss. Rather, it happened without a lot of thought. Most of the people who now accept the Steinmetz and Weickmann theory have never heard of Steinmetz and Weickmann. Until recently there were few photographs of odd radius halos and few photographs of pyramidal crystals. Although we now have ample numbers of photographs, there are not so many for which the radii of the halos or the interfacial angles of the crystals have been accurately measured. So although we think we are on the right track, we have no reason to be complacent.

Refraction Halos and Wedge Angle

Soon we will be looking at halos made by pyramidal crystals that are preferentially, rather than randomly, oriented. Crystals with plate orientations, Parry orientations, column orientations, and Lowitz orientations would all be examples of preferentially oriented crystals. When pyramidal crystals are preferentially oriented, many halos are possible—so many, in fact, that initially it may seem futile to come to grips with them. But the apparent complexity of the situation turns out to be a blessing in disguise, because it forces us to think more conceptually and systematically about halos in general. In the end it will turn out that the new halos are quite manageable. And what we learn will give insights into the old familiar halos as well.

Refraction halos are halos whose ray paths involve refraction only, with reflections playing no essential role. The 22° halo, the parhelia, and the tangent arc are examples of refraction halos, but the parhelic circle, the 120° parhelia, and the subparhelia are not, since their ray paths involve reflection.

Most of the common halos are refraction halos, and virtually all of the observed halos from pyramidal crystals are refraction halos. In subsequent chapters we will therefore concentrate on refraction halos. Even so, the situation may at first seem chaotic.

When the crystal orientations are not random

Let's try to get a sense of why preferentially oriented crystals might complicate matters. When crystals are randomly oriented they make circular halos, and that's all. But when the crystals are preferentially oriented, then for each circular halo that would have arisen in random orientations there is a whole family of related

arcs. Perhaps this is already clear from Chapter 6, where we had 22° plate, Parry, column, and Lowitz arcs all related to the 22° circular halo, and where we had 46° plate, Parry, column, and Lowitz arcs all related to the 46° circular halo. Nevertheless, this idea is so fundamental that it can stand some elaboration.

With randomly oriented crystals, you get circular halos, with each circular halo being associated with a wedge angle α of a crystal. If, for example, the crystals are hexagonal prisms, then wedges with $\alpha=60^\circ$ make the 22° halo. If the same crystals are preferentially oriented rather than randomly oriented, then each wedge with $\alpha=60^\circ$ will again make a halo. But whereas for random orientations any two wedges with $\alpha=60^\circ$ would make the same halo (the 22° halo), those same two wedges may now make different halos. Suppose, for example, that the crystals are falling with plate orientations. With the faces numbered as usual (Figure 6.2), wedge 35 would make the left parhelia and wedge 37 would make the right. These two wedges, which made the same halo when randomly oriented, now make different halos.

If instead these same crystals fall with Parry orientations, then wedges 35, 46, 48, and 57, all having wedge angle $\alpha=60^\circ$, make four different halos (Figure 6.6). If the crystals fall with column orientations, then the wedges with $\alpha=60^\circ$ make the tangent arc. And if they fall with Lowitz orientations they make three additional arcs.

Thus there is a whole family of refraction halos that have wedge angle $\alpha=60^\circ$. Those that arise in preferentially oriented crystals we refer to as 22° arcs. (More generally, all halos that are “arcs” arise in preferentially oriented crystals.) In the classification scheme of Figure 12.1, the 22° arcs would be the 22° plate arcs, the 22° Parry arcs, the 22° column arcs, and the 22° Lowitz arcs. In the sky, all 22° arcs are necessarily subsets of the annular region that is the 22° circular halo. The arcs tend to be weak unless they are fairly close to the inner boundary of the circular halo, and therefore the 22° arcs do indeed appear at an angular distance of 22° or a bit more from the sun.

For wedge angle $\alpha=90^\circ$ there is an analogous family of arcs—the 46° arcs—that appear 46° or a bit more from the sun. Some 22° and 46° arcs are shown in Figure 12.2.

So there is one 22° halo but many 22° refraction arcs, and there is one 46° halo but many 46° refraction arcs. When you recall that the pyramidal crystals of Chapter 8 can make eight different circular halos, and when you realize that each of those circular halos must have a corresponding family of refraction arcs, you begin to sense the challenge of understanding the halos made by preferentially oriented pyramidal crystals.

The challenge can be met, however, and in fact we have already begun to do so. It is clear from the preceding discussion that the wedge angle itself, or,

	$\alpha = 60^\circ$ $\Delta_{\min} = 22^\circ$	$\alpha = 90^\circ$ $\Delta_{\min} = 46^\circ$
plate orientations	22° plate arcs	46° plate arcs
Parry orientations	22° Parry arcs	46° Parry arcs
column orientations	22° column arcs	46° column arcs
Lowitz orientations	22° Lowitz arcs	46° Lowitz arcs
random orientations	22° halo	46° halo

FIGURE 12.1 Halo classification matrix. Each column is determined by a wedge angle α (60° or 90°), each row by a crystal orientation class (plate, Parry, column, Lowitz, or random). This matrix is for halos made by prismatic crystals; for pyramidal crystals the matrix would require eight columns instead of two. The matrix includes only refraction halos, that is, halos whose ray paths do not involve reflection.

FIGURE 12.2 (Left) Some 22° arcs, that is, refraction halos that have wedge angle $\alpha = 60^\circ$ and that arise in preferentially oriented crystals. All 22° arcs are subsets of the 22° circular halo, which is here indicated only by its inner and outer boundaries $\Delta = 22^\circ$ and $\Delta = 50^\circ$. (Right) Some 46° arcs. This diagram is the $\alpha = 90^\circ$ analogue of the one at left. See the tables of Chapter 6 for names of the individual arcs.

equivalently, the radius $\Delta_{\min}$ of the associated circular halo, provides a preliminary classification of refraction arcs. Just as there is a family of 22° arcs and a family of 46° arcs, there is also a family of 9° arcs, a family of 18° arcs, and so forth, one family for each of the circular halos from Chapter 8. The 9° arcs are 9° or a bit more from the sun, the 18° arcs are 18° or a bit more from the sun, and so forth.

Thus if you know the wedge angle that gives rise to a halo (assumed to be a refraction arc), then you know about how far from the sun to look for the halo. So the wedge angle is a fundamental entity, and indeed it has been recognized as such for centuries. But if you only know the wedge angle of the halo, there is obviously a lot that you still do not know about the halo. You do not know the shape of the halo, and you do not even know its location, since, although you know its approximate angular distance from the sun, you do not know its direction from the sun. Is there a fundamental entity of a halo that contains the information about shape and about direction from the sun, much as the wedge angle contains the information about angular distance from the sun? Yes, but it took a long time to see just what it was. In principle the missing information is contained in the wedge orientations that give rise to the halo. But how are you supposed to cope with wedge orientations? The beautiful and surprising answer is that you can do so in a simple and rather mindless way, without ever really focusing on the wedge orientations themselves. What is needed is the spin vector.

The Spin Vector

In spite of its esoteric name, the spin vector is simple. It will be the key to managing wedge orientations without actually having to think about them.

There is a spin vector associated with each of our four classes of non-random crystal orientations:

Plate orientations Recall that a hexagonal prismatic crystal is in plate orientation when face 1 is horizontal and on top (Figure 6.1). Thus the crystal is in plate orientation if $\mathbf{N}_1$, the outward normal vector to face 1, is vertical (and pointing up rather than down). The vector $\mathbf{N}_1$ is the spin vector for plate orientations. It is a vector which is fixed in the crystal and which characterizes plate orientations: The crystal is in plate orientation exactly when the spin vector is vertical.

The spin vector is illustrated with a wooden crystal model in Figure 13.1. The spin vector is the dowel, inserted in face 1. The crystal model is in plate orientation when the dowel points up.

Parry orientations A crystal is in Parry orientation when face 3 is horizontal and on top. Thus the crystal is in Parry orientation when $\mathbf{N}_3$, the normal vector to face 3, is vertical. The vector $\mathbf{N}_3$ is the spin vector for Parry orientations. It is a vector which is fixed in the crystal and which characterizes Parry orientations: The crystal is in Parry orientation exactly when the spin vector is vertical.

Column orientations A crystal is in column orientation when the crystal axis is horizontal, that is, when the vector $\mathbf{N}_1$ is horizontal. The vector $\mathbf{N}_1$ is the spin vector for column orientations. It is the same as the spin vector for plate orientations, but for column orientations the spin vector is horizontal, whereas

Wooden crystal model in plate orientation. The spin vector $\mathbf{P}$ (the dowel) is $\mathbf{N}_1$, the normal vector to face 1. In plate orientation the spin vector is vertical.

Crystal model in Parry orientation. The spin vector is $\mathbf{N}_3$ and is vertical.

Crystal model in column orientation. The spin vector is $\mathbf{N}_1$ and is horizontal

Crystal model in Lowitz orientation. The spin vector is $\mathbf{N}_1 \times \mathbf{N}_3$ and is horizontal.

FIGURE 13.1 Compare these photographs with Figure 6.1.

TABLE 13.1 Crystal orientation classes and spin vector.

Crystal orientation class	Spin vector P	Direction of P
plate orientations	N ₁	vertical
Parry orientations	N ₃	vertical
column orientations	N ₁	horizontal
Lowitz orientations	N ₁ × N ₃	horizontal

for plate orientations it is vertical. That is, the crystal is in column orientation exactly when the spin vector is horizontal.

Lowitz orientations Lowitz orientations are rare, and we mention them here mainly to illustrate further the concept of spin vector. A crystal is in Lowitz orientation when **N**₁ × **N**₃ (a vector parallel to face 1 and face 3) is horizontal. Hence the spin vector for Lowitz orientations is **N**₁ × **N**₃, and the crystal is in Lowitz orientation exactly when the spin vector is horizontal.

Thus for each of the above four classes of crystal orientations there is a vector **P**—the *spin vector*—which is fixed in the crystal and which characterizes the class of orientations: A crystal has the orientation in question exactly when the spin vector is vertical (plate and Parry orientations) or horizontal (column and Lowitz orientations).

Contact arcs and non-contact arcs

We said that the spin vector **P** is always horizontal or always vertical. Refraction arcs that arise in crystal orientations with **P** horizontal are called *contact arcs*, those that arise in crystal orientations with **P** vertical are called *non-contact arcs*. The middle diagram in Figure 5.5 can be thought of as showing the distribution of the spin vector for a contact arc, and the left-hand diagram as showing the same for a non-contact arc. In Figure 12.1 halos associated with the first and second rows—plate arcs and Parry arcs—are non-contact arcs, and those associated with the third and fourth rows—column arcs and Lowitz arcs—are contact arcs. The terminology of contact and non-contact has to do with whether the arc normally makes contact with the associated circular halo (Chapters 14 and 16).

Contact arcs behave quite differently from non-contact arcs. More importantly: In spite of our initial impression of chaos, all contact arcs behave alike, and all non-contact arcs behave alike. In the next chapter we will look at the non-contact arcs, then in Chapter 16 the contact arcs.

A User's Guide To Halo Poles

The concept of halo pole is crucial for understanding the many odd radius halos that can arise in preferentially oriented crystals. In this chapter we will try to tell enough about the concept so that it can be used in practice, but we won't say much by way of justification until Appendix F. For more of the logic behind the concept of halo pole, you should refer to the article by Tape and Können [77]. It was Können who originated the concept, in the 1990s.

Except for the circular halos, almost every halo has a so-called pole. If the halo is a non-contact arc, such as a plate arc or Parry arc, then the pole and the wedge angle together completely determine the halo. In fact, for a given sun elevation the qualitative appearance of the halo can be immediately inferred from the pole and wedge angle, and it can be done with little or no calculation. It is remarkable. And the same is true if the halo is a contact arc, such as a column arc or Lowitz arc: the pole and the wedge angle determine the halo, though not so simply as for non-contact arcs. It is therefore worth the effort to learn how to find halo poles, not just for odd radius halos but for halos in general.

It is no secret: The halo pole is just the spin vector as seen from the halo-making wedge. But we need a couple of preliminaries.

Unit sphere The unit sphere is a standard sphere of radius one. We use it to represent directions in space; a point on the unit sphere represents the direction of an arrow drawn to the point from the center of the sphere, as shown in Figure 14.1. We often opt not to distinguish between the point and the arrow. Thus the letter **V** in the figure refers either to the point or to the corresponding arrow.

We place a standard cartesian coordinate system at the center of the sphere, with the x -axis pointing to the front, the y -axis to the right, and the z -axis up.

FIGURE 14.1 Unit sphere. Each point V on the sphere represents a direction in space, namely, the direction of an arrow drawn to the point from the center of the sphere.

FIGURE 14.2 Halo-making wedge in standard orientation. The entry face of the wedge is toward the front of the sphere, the exit face to the rear.

Then the north pole of the sphere, for example, is the point $(0, 0, 1)$, and the equator is in the plane $z = 0$.

Wedge in standard orientation A halo-making wedge placed at the center of the sphere is said to be in *standard orientation* (Figure 14.2) when its entry and exit faces are oriented symmetrically with respect to the plane $x = 0$ that separates the front hemisphere from the rear, and when the entry face is toward the front, the exit face is toward the rear, and the wedge opens downward.

To find halo poles

The pole of a halo is a certain point on the unit sphere. The pole can be defined and calculated analytically, but we prefer here a less formal approach. As a first example we consider the circumzenith arc made by crystals having Parry orientations (Figure 6.6). The ray path for the halo enters face 3 (the top prism face) of the crystal and exits face 1 (a basal face). We recommend that you construct a model of the crystal, number the faces as usual, and insert a nail or dowel part way into face 3 to represent $\mathbf{N}_3$, the spin vector $\mathbf{P}$ for Parry orientations (Figure 13.1). Then to get the pole of the halo, you simply orient the crystal model so that wedge 3 1 is in standard orientation. When the wedge is in standard orientation the head of the nail (the spin vector $\mathbf{P}$) points in the direction of the pole of the halo. In this example the pole is therefore as shown in Figure 14.3; it is on the meridian ($y = 0$) of the sphere and 45° above the equator, since the wedge angle is 90° .

FIGURE 14.3 The pole of the circumzenith arc that arises in Parry orientations. The halo-making wedge is wedge 31, and the spin vector $\mathbf{P}$ is $\mathbf{N}_3$. When the wedge is in standard orientation, as here, the spin vector $\mathbf{P}$ points in the direction of the halo pole. The left-hand diagram shows the entire crystal, whereas the right-hand diagram, in order to focus attention on the halo-making wedge, shows only the wedge.

FIGURE 14.4 The pole of the halo that arises in Parry orientations and that has wedge 81. See text. The halo is the upper right 46° Parry arc.

For a second example of a halo pole, consider the halo that arises in crystals having Parry orientations and that has ray path 81. The only difference between this example and the previous one is that the ray path is now 81 rather than 31. The spin vector $\mathbf{P}$ is therefore $\mathbf{N}_3$ as before, but now it is wedge 81 that must be put in standard orientation in order to find the pole of the halo. So start with your crystal model oriented as in Figure 14.3, with wedge 31 in standard orientation as before, and then rotate it 60° about its c -axis. The wedge 81 moves into standard orientation, and the point $\mathbf{P}$ moves along the great circle shown in Figure 14.4, eventually arriving at the position shown there—the pole of the halo.

The preceding halos are two of several 46° Parry arcs, that is, refraction arcs that have wedge angle $\alpha = 90^\circ$ and that arise in crystals having Parry orientations.

The poles of the 46° Parry arcs are shown in Figure 14.5. You can verify the pole locations using your crystal model as above; just treat each of the 90° wedges in succession. You will find, for example, that the Parry arc with ray path 15 has the pole shown at the lower right of the sphere; that is the direction of the nail when your model is oriented so that wedge 15 is in standard orientation.

How would the procedure change if the halos were plate arcs instead of Parry arcs? Only in that the spin vector $\mathbf{P}$ would then be $\mathbf{N}_1$, the outward normal to face 1, instead of $\mathbf{N}_3$. So you would need to move the nail to face 1 of your crystal model. The pole of the right parhelion, for example, would turn out to be the point $(0, 1, 0)$ at the far right of the sphere.

Perhaps you can see from these examples that the pole of the halo is just a convenient means of specifying the wedge orientations that are responsible for the halo. That is why the pole, together with the wedge angle, determines the halo.

To infer the appearance of a non-contact arc from its pole

We consider as examples the 46° Parry arcs, Figure 14.5. These halos are rare, but they are more akin to the odd radius halos in the next chapter than are the more common halos that we could have considered instead, and this is why we have chosen them as illustrations. The figure shows their poles, together with a point of the unit sphere marked $\mathbf{D}_u$, the so-called minimum deviation (entry) point. The point $\mathbf{D}_u$ is on the meridian ($y = 0$) at an angle $\Delta_{\min}/2 = 23^\circ$ below the equator.

The figure also shows the halos themselves. Remarkably, the direction of each halo from the sun is nearly the same as the direction of its pole from the point $\mathbf{D}_u$. And since the wedge angle is 90° , the halos are all 46° or a bit more from the sun. So from the pole diagram and the wedge angle you can see at once where to look in the sky for each halo.

The halos in Figure 14.5—the 46° Parry arcs—are examples of non-contact arcs. The close relation between the poles and the halos holds not just for the halos in the figure but for non-contact arcs in general. The location of the pole of a non-contact arc therefore gives a natural way to classify and describe the arc. Thus the arc is a *left* arc or *right* arc according to whether its pole is on the left or right hemisphere. The arc is *symmetric* if its pole is on the meridian ($y = 0$) of the sphere, because it turns out that such halos are indeed left-right symmetric. The arc is an *upper* arc or *lower* arc according to whether its pole is above or below the great circle passing through the points $\mathbf{D}_u$ and $(0, \pm 1, 0)$ (Figure 14.6). Upper left and right arcs are also called supralateral arcs, and lower left and right arcs are called infralateral arcs.

We said that from the pole diagram you can tell where the halos appear in the sky. You can also anticipate the shapes of the halos, at least if the tilts of

FIGURE 14.5 (Top left) Poles (black dots) of 46° Parry arcs. Each pole is a point on the unit sphere. The minimum deviation point D_U is located $\Delta_{\min}/2$ below the equator as shown. (Top right) Same but in stereographic projection looking down the x -axis. (Bottom) The 46° Parry arcs themselves. The direction of each arc from the sun is about the same as the direction of its pole from D_U . Depending on sun elevation Σ , an arc can be empty, that is, it does not appear. For $\Sigma = 20^\circ$ (left) the lower symmetric arc (i.e., the circumhorizon arc) is empty, whereas for $\Sigma = 70^\circ$ (right) the other five arcs are empty.

FIGURE 14.6 Terminology for non-contact arcs. Each non-contact arc is classified according to the location of its pole on the unit sphere. The dividing curve between right and left arcs is of course the meridian $y=0$, but the dividing curve between upper and lower arcs is the great circle passing through the points $\mathbf{D}_u$ and $(0, \pm 1, 0)$ (not the equator $z=0$). The suncave arcs are those having poles close to $\mathbf{D}_u$, and the sunvex arcs are those having poles far from $\mathbf{D}_u$.

the crystals are small. If a non-contact arc has its pole fairly far from both the minimum deviation point $\mathbf{D}_u$ and its antipode $-\mathbf{D}_u$, then, when the arc is non-empty, it will be a more or less U-shaped curve convex toward the sun; such arcs are referred to as *sunvex*. On the other hand, if the arc has its pole close to $\mathbf{D}_u$ (or $-\mathbf{D}_u$), then, when it is non-empty, it will be concave toward the sun; such arcs are *suncave*. Thus the poles of sunvex arcs occupy a broad and vaguely defined band of the sphere about 90° from $\mathbf{D}_u$ —this is the region far from $\mathbf{D}_u$ and $-\mathbf{D}_u$ —while the poles of suncave arcs occupy the region close to $\mathbf{D}_u$ (Figure 14.6). An example of a sunvex arc would be the upper right 46° Parry arc, and an example of a suncave arc would be the lower symmetric 46° Parry arc, both shown in Figure 14.5.

There is more. Again we use the 46° Parry arcs as examples. Each of these arcs, as we said, is on or just outside the 46° halo. But for each there is just one sun elevation Σ for which the arc actually contacts the 46° halo. The zenith angle σ of the sun at that moment is the same as the angular distance s from $\mathbf{D}_u$ to the pole $\mathbf{P}_u$ of the arc; that is, $\sigma = s$, and therefore $\Sigma = 90 - s$. And the direction of the contact point from the sun is exactly the direction of $\mathbf{P}_u$ from $\mathbf{D}_u$. This is illustrated in Figure 14.7 for the upper right 46° Parry arc.

A much easier example would be the upper symmetric 46° Parry arc, whose pole we found in Figure 14.3. Can you find the sun elevation for contact and then the location of the contact point on the 46° halo? Hint: Figure 14.5.

Because for any non-contact arc the angular distance s from $\mathbf{D}_u$ to $\mathbf{P}_u$ gives the solar zenith angle for which the arc contacts the circular halo, it also gives an indication of the solar zenith angles for which the arc is non-empty, that is, for which it can appear. Roughly: the arc is non-empty for solar zenith angles close to s and is empty for solar zenith angles far from s . Thus a suncave arc will appear when the sun is high overhead, since its pole is close to $\mathbf{D}_u$, whereas a

FIGURE 14.7 Upper right 46° Parry arc, showing the close relation between its pole and its contact point with the 46° halo. (Left) The pole P_u , viewed looking directly down at the minimum deviation point D_u from outside the unit sphere. The angular distance s of P_u from D_u is 79° . (Middle) Showing the arc for the one sun elevation $\Sigma = 90 - s = 11^\circ$ for which it contacts the 46° halo. The direction of the contact point H from the sun S is exactly the same as the direction of P_u from D_u . This relation holds for all non-contact arcs. (Right) Same but for $\Sigma = 22^\circ \neq 11^\circ$. The arc does not quite contact the circular halo.

sunvex arc will appear when the sun is low, since its pole is far from D_u . But the wedge angle also plays a role, since halos arising from small wedge angles tend to be visible over a wider range of sun elevations than those arising from larger wedge angles.

From the preceding paragraph you can see that a non-contact arc with pole far to the rear of the sphere can only appear when the sun—or other light source—is below the horizon. Such a situation might arise when sunlight reflects from a frozen lake surface and then encounters ice crystals in the atmosphere; the light source is then the sun's reflected image, which is below the horizon. Such situations are obviously unusual, and in this book we ignore non-contact arcs whose poles are far to the rear of the sphere.

Much of what we said in this chapter depends on the tilts of the crystals being very small. In practice some qualification is needed, since small tilts are the exception rather than the rule. With even moderate tilts, non-contact arcs will contact the circular halo for a broad range of sun elevations, rather than at just one elevation. And for moderate to large tilts, the arcs tend to become circular, so that the distinction between sunvex and suncave is lost.

Reminder: In this section we have been talking about non-contact arcs. Contact arcs will be treated later.

Odd Radius Plate Arcs

Odd radius plate arcs arise in pyramidal crystals having plate orientations. These arcs are rare, but not so rare as you may imagine. They are seen much more often than odd radius circular halos, for example.

Mostly we will restrict the discussion to refraction arcs, that is, to arcs involving refraction only; these halos are by far the most important. And as in Chapter 8, where we studied odd radius circular halos, the crystals that we will use to explain the odd radius plate arcs are pyramidal crystals with $\{10\bar{1}1\}$ faces (Figure 15.1). As explained in Chapter 12, the plate arcs resulting from these crystals fall into eight families—the 9° plate arcs, the 18° plate arcs, the 20° plate arcs, the 22° plate arcs, and so forth—one family for each of the circular halos of Table 8.1. Here we are including the familiar 22° and 46° plate arcs; although they are not, strictly speaking, odd radius, they do sometimes form in pyramidal crystals.

FIGURE 15.1 Pyramidal crystal with plate orientation, that is, with basal faces horizontal. The pyramid faces here are the $\{10\bar{1}0\}$ faces, the same as for the circular halos considered in Chapter 8.

First let's look at some photographs of odd radius plate arcs. Later we will explain in more detail why we think we know what we are seeing in such photographs.

At first glance the halo display in Figure 15.2 appears quite ordinary—just a pair of bright but ordinary parhelia. But had you been present at the time of the display, and had you used the extended-arm-and-spread-fingers method of Chapter 1 to estimate the angular distance of these arcs from

FIGURE 15.2 The 18° plate arcs. Fairbanks, Alaska, March 3, 1999. $\Sigma = 14^\circ$.

FIGURE 15.3 Same display as Figure 15.2 but photographed thirteen minutes earlier. The two inner arcs are 18° plate arcs, and the outer arc is probably a halo intermediate between a 23° plate arc and a 23° circular halo. The colored dots are at $\Delta_{\min} = 18.3^\circ, 21.8^\circ$, and 22.9° from the sun (Table 8.1).

the sun, you would have found that they were too close to the sun to be parhelia. And had you been present ten minutes earlier, you would have seen the same pair of bright arcs but accompanied by a nearly circular arc about five degrees further from the sun (Figure 15.3), making it obvious that some, at least, of the halos in this second figure must be odd radius. In fact, the two inner arcs are 18° plate arcs, and the outer arc is probably something intermediate between a 23° plate arc and a 23° circular halo.

Although that outer halo in Figure 15.3 may look like a 22° halo, it is not: Using the methods of Appendix D, we have positioned colored dots at the indicated angular distances from the sun. The green dot, 21.8° from the sun ($\Delta_{\min}$ for the 22° halo), is distinctly sunward of the halo in question; the halo cannot be the 22° halo. However, both in this figure and in the previous, pieces of what is probably the 22° halo do appear in the low crystal swarm just above the horizon.

Figure 15.4 shows a lunar display in which a number of bright stars happen to be located near the halos. Since the time and place of the photograph are known, the angular distances from the various stars to the moon can be calculated

FIGURE 15.4 Lunar halo display with 9° , 18° , and 23° plate arcs. The angular distance from the moon to each circled star is given. Fairbanks, December 5, 2003, 10:40 p.m. Moon elevation 41° .

accurately. From these angular distances it is evident that all of the halos in the photograph are odd radius. Even the bright arc at the top, which resembles the ordinary upper tangent arc (i.e., the 22° column arc) is an odd radius arc; the upper tangent arc would be $\Delta_{\min} = 21.8^\circ$ above the moon, but the arc in the photo is nearly 23° above the moon—it is the upper 23° plate arc.

The 18° plate arcs are also present, to the left and right of the moon, and the lower 9° plate arc is present below the moon. Here these arcs appear as enhancements of their corresponding circular halos rather than as well-defined curves in their own right. This is typical; only in the best displays do odd radius plate arcs approximate the distinctive curves that would be expected from precisely horizontal crystals.

Figure 15.5 is a daytime display just a bit more complex than Figure 15.4. You can see immediately that at least some of the halos must be odd radius. The

FIGURE 15.5 Halo display with odd radius plate arcs. Some circular halos are present as well, including the 20° halo, just outside the 18° plate arc. Vaala, Finland, May 12, 2002. $\Sigma = 29^\circ$.

enhancements at various positions on the circular halos are signs that there is more here than just circular halos. The enhancements are plate arcs, namely, the lower 9°, the right 18°, the upper 23°, and the lower right 24° plate arcs.

Organizing the plate arcs

Figure 15.6 shows the theoretically expected plate arcs for sun elevations $\Sigma = 20^\circ$ and $\Sigma = 70^\circ$. Nearly all of the halos in the simulations are refraction arcs, and since they are plate arcs, they are non-contact arcs. They can therefore be understood in the context of the halo pole theory of the preceding chapter. Figure 15.7 is the relevant pole diagram; it shows the pole for each plate arc. From the pole and $\Delta_{\min}$ we know what to expect for the corresponding arc, and it is therefore easy to correlate the poles in the pole diagram with the arcs in the simulation: The direction of the pole from the point $\mathbf{D}_u$ gives the approximate direction of the arc from the sun, and $\Delta_{\min}$ gives the approximate angular distance of the arc from the sun. If the pole is far from $\mathbf{D}_u$, then the arc is sunvex and will appear when the sun is low. If the pole is close to $\mathbf{D}_u$, then the arc is suncave and will appear when the sun is high. The lower 23° arc (wedge 13 2), for example, having its pole very close to $\mathbf{D}_u$, appears only in the $\Sigma = 70^\circ$ simulation and is highly suncave. The upper 9° arc (wedge 13 6) and the two upper 24° arcs (13 5 and 13 7), having their poles fairly far from $\mathbf{D}_u$, appear only in the $\Sigma = 20^\circ$ simulation and are sunvex. The upper 20° arc and the two upper 35° arcs, with poles even farther from $\mathbf{D}_u$, appear in neither simulation; they would show up at very low sun elevation.

Simulations and halo poles are complementary approaches to understanding halos. Each approach does some things that the other cannot do. One of the things that the halo pole theory does is to exhibit all of the possibilities at once, in a single diagram. In the present case, all plate arcs are given in the pole diagram of Figure 15.7.¹ This is something that no single simulation by itself can do. On the other hand, although the pole and the $\Delta_{\min}$ -value of a non-contact arc do in principle determine the arc, it is a highly idealized arc that they determine. Simulations give much more realistic results, as we will see momentarily.

Simulations of some real displays

Figure 15.8 is a first attempt at simulating the halo display of Figure 15.2. The crystals used in making the simulation were plate-oriented pyramidal crystals shaped like the one shown. They were given very small tilts, which resulted

¹ Recall that we are talking about refraction arcs, we are assuming that the pyramidal crystal faces are the $\{10\bar{1}1\}$ faces, and we are ignoring arcs with poles on the rear hemisphere.

FIGURE 15.6 Odd radius plate arcs. These simulations were made using plate-oriented pyramidal crystals shaped like the one in Figure 15.1. The tilts of the crystals were about 1° . Each refraction arc is labeled with its $\Delta_{\min}$ -value, that is, with the radius of the corresponding circular halo. The true angular distance of an arc from the sun may be somewhat more than $\Delta_{\min}$, as in the left-hand simulation, where the 23° arc is actually about 28° from the sun. The tick marks, in a line emanating from the sun, are at one-degree intervals. (Left) $\Sigma = 20^\circ$. (Right) $\Sigma = 70^\circ$.

FIGURE 15.7 Poles of plate arcs. The responsible crystals have prism, basal, and $\{10\bar{1}1\}$ pyramid faces as usual. The pole and $\Delta_{\min}$ determine the arc, as explained in the text; compare this figure with Figure 15.6. The minimum deviation point D_u varies with $\Delta_{\min}$, but it is confined to the small elongated region marked " D_u ." (Left) Each pole is labeled with its $\Delta_{\min}$ -value. (Right) Each pole is labeled with its wedge.

FIGURE 15.8 A first attempt to simulate the halo display of Figure 15.2. The crystals used in making the simulation were shaped like the one shown and had plate orientations with very small tilts. The diagram at upper right depicts the crystal orientations, as explained in Figure 5.3.

FIGURE 15.9 Same as Figure 15.8 but with larger crystal tilts. Large crystal tilts seem to be the norm for odd radius displays; odd radius arcs as well defined as those in Figure 15.8 are rare.

in unusually well-defined halos. Using the pole diagram of Figure 15.7, we can identify the halos in the simulation; they must be the left and right 18° plate arcs and the upper 23° plate arc. The direction of each arc from the sun is the same as the direction of its pole from $\mathbf{D}_u$, as expected. The angular distance of the 18° arcs from the sun is close to 18° , also as expected. And although the angular distance of the 23° arc from the sun is considerably more than 23° , this could have been anticipated, since the sun elevation for contact with the 23° circular halo turns out to be 48° —substantially different from the sun elevation of the display.

Why don't the remaining arcs with poles in Figure 15.7 appear in the simulation in Figure 15.8? Since there are no prism faces on the crystal, there are no 9° , 22° , 24° , or 46° arcs. The upper 20° plate arc is absent partly because its pole is far from $\mathbf{D}_u$, so that the arc requires very low sun, and partly because the relatively large basal face 1 of the crystal interferes with the necessary ray path 13 16. The upper 35° plate arcs are absent because their poles are too far from $\mathbf{D}_u$, and the lower 20° and 35° plate arcs are absent because they are below the horizon at the sun elevation of this display. (In low level displays in Fairbanks we do sometimes see the lower 35° plate arcs below the horizon, showing up as concentrations of sparkles between ourselves and the ground, sometimes in the space of only a few meters.) The lower 23° plate arc has pole close to $\mathbf{D}_u$ and therefore occurs only at high sun.

The sorting of ray paths shown in the figure confirms that the halos in the simulation are mainly the 18° plate arcs and the upper 23° plate arc. Also present is a faint odd radius helic arc, formed by reflections from pyramid faces.

In Figure 15.9 the tilts of the crystals have been increased. The 18° plate arcs are no longer dramatically sunvex and instead look more like short pieces of the 18° circular halo. The 23° plate arc has become so diffuse and spread out as to nearly lose its identity. There is also a hint here of the upper 20° plate arc, and this is confirmed by a sorting of ray paths (not shown), which finds a few 13 16 ray paths—these paths are more realizable now, due to the larger tilts of the crystals.

Comparison of the simulation with the photograph of the display (Figure 15.2) makes us think we understand the display fairly well: We are seeing mainly the 18° plate arcs together with some diffuse and weak light that is the upper 23° plate arc.

The display in Figure 15.10 is more complex. The 18° and 23° circular halos are present, and the 20° circular halo is probably there as well, blending with the 18° halo. But there are plate arcs also, some of them quite subtle. The 18° and (probably) 22° plate arcs are bright and well defined, but the 24° plate arcs, near the purple dots, are fainter and more diffuse, and the 20° plate arc is fainter still. The 9° plate arc is not clearly distinguishable from the sun pillar. All of the plate arcs are where Figure 15.7 predicts.

FIGURE 15.10 Halo display with odd radius plate arcs. The colored dots are at angular distances of 18.3°, 21.8°, and 23.8° from the sun (Table 8.1). In the simulation, crystals like the upper one made the 9°, 18°, 20°, and 24° plate arcs, crystals like the middle one made the 22° plate arcs, and crystals like the lower one made the circular halos. Fairbanks, November 15, 2003. $\Sigma = 4^\circ$.

FIGURE 15.11 Lunar display with odd radius plate arcs. The simulation was made using plate oriented crystals like the one shown. The yellow and red dots in the photo are 32° and 34.9° from the moon; like the yellow and red dots in Figure 8.3, they argue in favor of the Steinmetz and Weickmann value $x = 28^\circ$ over the Besson and Humphreys value $x = 25^\circ$; see the discussion on page 121. The asymmetry of the halos is not real but results from the photo not being centered on the moon. Fairbanks, 4:19 a.m., January 6, 2004. Moon elevation 35° .

The simulation is a fairly good match for the photo, but it is not perfect. The 9° plate arc in the simulation is too strong, and the 24° plate arcs do not seem to be shaped quite right. This is fairly typical of our simulations of odd radius halo displays. That is, the simulations tend to be good, but not perfect. It makes us a bit nervous.

The display in Figure 15.11 has 9°, 18°, 20°, 23°, 24°, and 35° plate arcs. Several stars are identified in the photo, and their angular distances from the moon are given. The location of the star TYC 3391-2638-1 shows that the arc at the top of the photo is indeed the upper 23° plate arc rather than the upper tangent arc; the arc is too far from the moon to be the tangent arc. And the location of α Ori shows that the concentration of light at the bottom of the photograph is indeed the lower 20° plate arc rather than the lower tangent arc.

There are no stars visible near the 35° plate arc to confirm its angular distance from the moon. We have therefore positioned the red dot so that its angular distance from the moon is 34.9°—precisely the theoretical radius of the 35° halo given in Table 8.1. Although the 35° arc here is a plate arc rather than a circular halo, the tilts of the crystals are large enough so that the 35° arc would have been in contact with the circular halo had it been present. Hence, since the red dot is on the inner edge of the arc, this display gives good empirical support for the theoretical value $\Delta_{\min} = 34.9^\circ$ for the 35° halo radius.

For comparison, a yellow dot is positioned 32° from the moon. To understand the significance of this second dot, recall from Chapter 11 that historically the correct value for x has been in doubt, with $x = 28^\circ$ (Steinmetz and Weickmann) and $x \approx 25^\circ$ (Besson and Humphreys) being two competing values for x . For the halo under consideration here, the value $x = 28^\circ$ gives the theoretical value $\Delta_{\min} = 34.9^\circ$ of the preceding paragraph, whereas $x = 25^\circ$ gives $\Delta_{\min} = 31.9^\circ$. Thus the location of the halo with respect to the two dots argues strongly for the Steinmetz and Weickmann value $x = 28^\circ$. We came to the same conclusion on page 121 in connection with Figure 8.3.

Halo associations

Whereas in Figure 15.8 the odd radius plate arcs were the 18° and 23° arcs, in Figures 15.12 and 15.13 they are mainly the 9° and 24° arcs. These halo associations are common; the 18° and 23° arcs tend to occur together, and the 9° and 24° arcs tend to occur together. These associations are not limited to plate arcs; Figure 8.4, for example, illustrates the 18°–23° association for circular halos.

You can begin to understand halo associations by comparing the pyramidal crystal in Figure 15.8 with the pyramidal crystals in Figures 15.12 and 15.13.

FIGURE 15.12 Halo display with upper 9° plate arc—the bright spot above the sun. Also visible are bright 22° plate arcs at left and right (i.e., ordinary parhelia), a faint 22° circular halo, and faint upper 24° plate arcs at 2:00 and 10:00 just outside the 22° halo. Crystals like the lower one made the plate arcs in the simulation, and crystals like the upper one made the 22° halo. Fairbanks, February 26, 2000. $\Sigma = 7^\circ$.

FIGURE 15.13 Lunar display with lower 9° and lower 24° plate arcs. The lower 9° plate arc is the bright spot below the moon, and the lower 24° plate arcs are the areas of illumination at 4:00 and 8:00 just outside the 22° circular halo. Crystals like the lower one made the plate arcs in the simulation, and crystals like the upper one made the circular halos. Fairbanks, 6:04 P.M., January 5, 2004. Moon elevation 25°.

Prism faces are necessary for the ray paths for the 9° and 24° arcs, but large prism faces interfere with the ray paths for the 18° and 23° arcs. There is more to it than that, however, and the halo associations do not invariably materialize.

The 23° plate arc

The lovely halo in Figure 15.14 was photographed by Timo Viinanen in Finland in August of 2004. Viinanen did well to recognize that the display was special and then to get a sequence of photos documenting its evolution over the course of an hour and a half. The bright halo shown here was by far the most prominent halo of the display, but faint odd radius halos were also present from time to time. The most conspicuous of the faint halos seem to have been 18° plate arcs.

We believe that the halo in the photo is the upper 23° plate arc. The identification is not automatic, however, since theoretically the upper 23° plate arc barely differs in appearance from the upper suncauve Parry arc. The two arcs are shown in the simulations in Figure 15.15. It is not surprising that the arcs look alike, since they have nearly the same wedge angle and nearly the same pole.

What, then, are we seeing in the photograph—a Parry arc or an odd radius plate arc? Halo displays that contain unequivocal Parry arcs almost always contain well developed column arcs as well, as in Figures 1.8, 6.7, and 6.8, so the absence of column arcs in the photo favors the odd radius plate arc identification. So does the presence of the 18° plate arcs in the display.

That is about as far as we can go, without more information. For the sun elevation in question the Parry arc is theoretically about a degree closer to the sun than is the plate arc. If the sun had been in the photo, then careful measurements might have distinguished between the two possibilities. And had polarization observations been made during the display, as described briefly in Appendix C and in much more detail by Können [40], they might also have been decisive.

Both explanations—Parry arc and odd radius plate arc—require some rationalizing regarding the halo associations predicted by the simulations. According to the simulations, either the lower 20° plate arc or the lower sunvex Parry arc should probably have been visible, though faint. Viinanen did not see them, so we have to believe that the halo-making cloud did not extend into the relevant region, or that the cloud was too thick there to make a halo. (The other, unlabeled halos in the simulations would perhaps have been too faint to be seen in reality.)

Figure 15.16 is a similar display, but with $\Sigma = 22^\circ$. The photographer, Les Cowley, analyzed the display and concluded that this halo, too, was probably the upper 23° plate arc. Some of his analysis can be found on his website; see the footnote on page 51.

FIGURE 15.14 Probable upper 23° plate arc. Hämeenlinna, Finland, August 8, 2004. $\Sigma = 41^\circ$. Photo © Timo Viinanen.

FIGURE 15.15 Comparison of upper 23° plate arc (*left*) and upper suncave Parry arc (*right*). The simulations were made with the indicated crystals—plate oriented pyramids for the left, and Parry oriented columns for the right. The simulations include much more sky than in the photograph above. $\Sigma = 41^\circ$.

FIGURE 15.16 Probable upper 23° plate arc. Norfolk, England, July 5, 2000. $\Sigma=22^\circ$. Photo © Les Cowley.

The remarkable brightness and quality of the halos in these two photographs raises the question of whether we might be missing the boat entirely. Could these halos be instances of some new halo altogether—neither Parry arc nor 23° plate arc? We do not think so, but we can't be positive.

More on the upper 23° plate arc can be found in an article by Riikonen et al [61]. They believe that upper 23° plate arcs are more common than has been thought, and that many of them have been mistaken for upper tangent arcs. It is easy to see how this could happen, especially if the arc were of only mediocre quality, like the one in Figure 15.4, for example.

An open problem

We do not understand the two very similar displays shown in Figures 15.17 and 15.18. In each photo the two bright arcs closest to the sun look just right for the 18° plate arcs 13 25 and 13 27, and we can easily simulate them. In particular, it is quite all right that the arcs are farther than $\Delta_{\min} = 18.3^\circ$ (orange dots) from the sun, since the sun elevation here is not close to the theoretical sun elevation 31° needed for contact, and since the arcs show at least a hint of sunvex shape,

FIGURE 15.17 Odd radius plate arcs? The bright inner arcs are probably the 18° plate arcs, but the fainter arcs just outside them are harder to identify. See text. The orange and green dots are 18.3° and 21.8° from the sun, the $\Delta_{\min}$ -values for the 18° and 22° halos. Fairbanks, January 25, 1996. $\Sigma = 6^\circ$. 20mm lens.

FIGURE 15.18 Similar. Fairbanks, February 2, 1996. $\Sigma = 6^\circ$. 28mm lens.

suggesting that the tilts of the crystals are not large. But the identity of the two outer arcs is in doubt. They might be the 22° plate arcs, they might be the 18° plate arcs 23 15 and 23 17,² or they might be neither. We are unable to make a simulation that is completely satisfactory. Is our failure to simulate the display a consequence only of a missing detail, or does it represent some fundamental misconception? We do not know. The fact that such similar displays occurred on two different days suggests that the circumstances were not especially pathological and that a simulation should not require a lot of tweaking of crystal shapes and orientations. But we have had no luck.

Finding the halo poles

Using the method of Chapter 14, you can verify the pole locations shown in Figure 15.7 for yourself, if you are willing to build a model of a standard pyramidal crystal (Appendix A). Since the crystal orientations are plate orientations, then the spin vector $\mathbf{P}$ is $\mathbf{N}_1$, the normal vector to face 1. If you want the pole of the plate arc due to, say, wedge 13 27, you just put wedge 13 27 in standard orientation; the vector $\mathbf{P} = \mathbf{N}_1$ then points in the direction of the pole. This is illustrated in Figure 15.19, but the figure is a poor substitute for a crystal model. With the model in your hands, you will easily see whether the relevant wedge is in standard orientation. This is not so clear from the figure.

The remaining pole locations can be verified in the same way, with the crystal model. Remember that this is more than amusement. By finding the poles, you are finding all (refraction) plate arcs that result from standard pyramidal crystals.

FIGURE 15.19 Finding the pole of the plate arc with wedge (ray path) 13 27. The crystal is oriented so that wedge 13 27 is in standard orientation. The spin vector $\mathbf{P} = \mathbf{N}_1$ then points in the direction of the pole. This same pole appears in Figure 15.7.

² Their poles are on the rear hemisphere and not seen in Figure 15.7.

A look at the crystals

This chapter concludes with Figures 15.20 and 15.21, showing an odd radius halo display and some of the crystals that made it. Many of the crystals were pyramidal, as expected.

Perhaps it is a good time to recall that pyramidal ice crystals are barely known in the scientific literature and that, where known, they are often regarded as impossibly rare. But every odd radius halo display, like the one in Figure 15.20, betrays the presence of countless such crystals. At such moments pyramidal ice crystals are far from rare.

FIGURE 15.20 Odd radius lunar display. Some of the crystals that fell during the display are shown in Figure 15.21. Fairbanks, December 24, 2004. $\Sigma = 39^\circ$.

FIGURE 15.21 ESEM photos of some crystals collected during the display of Figure 15.20. The black and white scale bars on the photos, clockwise from upper left, are 30, 50, 50, 50, 45, and 100 μ . The selection of photos exaggerates the frequency of pyramidal crystals in the crystal sample, but not by much. Photos © Kenneth P. Severin.

Odd Radius Column Arcs

Odd radius column arcs arise in pyramidal crystals having column orientations. Such halos are nearly unknown in the halo literature. They are indeed rare, but they are not so rare as the literature suggests, since they can be mistaken for pieces of circular halos, in which case their significance is missed. Halos that appear at first glance to be circular should always be scrutinized to see whether they might be column arcs.

As in previous chapters we tentatively assume in this chapter that the pyramidal crystals are the usual ones, that is, that their pyramid faces are the $\{10\bar{1}1\}$ faces. Figure 16.1 shows such a crystal in column orientation, and Figure 16.2 shows the halos that such crystals would theoretically be expected to make if the orientations were nearly perfect, that is, if the tilts of the crystals were nearly zero. The halos are therefore column arcs, though highly idealized. We should say at the outset that we have never seen a real display as lovely and elaborate as that in the figure. But idealized displays like this one are easier to understand than real

displays, in which large crystal tilts and diffraction effects can smear out the halos and make them harder to recognize. We therefore postpone the real displays for the time being.

How, then, are we to interpret the display in Figure 16.2? Nearly all of the halos will turn out to be refraction arcs. As usual, they can be crudely classified according to the radius $\Delta_{\min}$ of the associated circular halo. That

FIGURE 16.1 Pyramidal crystal with column orientation—the c -axis is horizontal. The pyramid faces are the $\{10\bar{1}1\}$ faces.

FIGURE 16.2 Odd radius column arcs. The crystals used in making this simulation were shaped like the one in Figure 16.1. They were given nearly perfect column orientations, that is, their c -axes were almost perfectly horizontal. Individual arcs will be identified later. The angular width of the simulation is 80° . $\Sigma = 20^\circ$.

is, the halos should fall into eight families—the 9° column arcs, the 18° column arcs, the 20° column arcs, and so forth.

This classification, however, does not go far in explaining the display. The halos still seem frighteningly complex, with no apparent sense to their shapes. In spite of this apparent complexity, however, there is an underlying order—a simplicity, even—that is encoded in their poles. And the poles are easy to find. The poles for these halos—column arcs—are the same as the poles for plate arcs, since the spin vector is the same for both (Table 13.1). The poles are therefore exactly as shown in Figure 15.7.

Column arcs are contact arcs, whereas plate arcs are non-contact arcs. Like non-contact arcs, contact arcs are determined by their poles and $\Delta_{\min}$ -values. The $\Delta_{\min}$ -value of a contact arc of course tells you the approximate angular distance of the arc from the sun, just as it does for non-contact arcs. But what does the pole of a contact arc tell you? The answer is quite different from that for non-contact arcs.

To infer the appearance of a contact arc from its pole

Recall that each non-contact arc makes contact with its corresponding circular halo for only one sun elevation Σ , namely, $\Sigma = 90 - s$, where s is the angular distance between the halo pole $\mathbf{P}_u$ and the minimum deviation point $\mathbf{D}_u$. Each contact arc, however, makes contact with its circular halo for a whole range of sun elevations Σ , namely, $-s \leq \Sigma \leq s$. For each sun elevation between $-s$ and s there are exactly two contact points with the circular halo. Their configuration on the circular halo changes with sun elevation as illustrated in Figure 16.3. The configuration is simplest when $\Sigma = 0$; in that case each contact direction is perpendicular to the pole direction, and the two contact points are therefore at diametrically opposite locations on the circular halo. As Σ increases, the two contact points move along the circular halo in the direction opposite to the pole direction, thus moving closer together, until at $\Sigma = s$ the two points merge into a single point.

Contact arcs whose poles are not too close to $\mathbf{D}_u$ will be concentrated near their contact points so long as the sun is fairly low. In such cases the contact points tell a lot about the halo. The $\Sigma = 0^\circ$ and $\Sigma = 20^\circ$ diagrams in Figure 16.3 are good illustrations. However, as the sun elevation increases, the halos tend to become circular, and the contact points become less relevant, as in the $\Sigma = 40^\circ$ diagram. Contact points are also less relevant for contact arcs with poles close to $\mathbf{D}_u$, since such arcs tend to be nearly circular for all sun elevations.

It turns out that each contact arc has two poles, located at antipodal points of the sphere. (Requiring that $\mathbf{P}$ be horizontal is the same as requiring that $-\mathbf{P}$ be horizontal.) When using the pole direction to estimate the contact directions, one should always choose the pole that is closer to $\mathbf{D}_u$. Thus, for example, in Figure 15.7, the poles 13 17 and 23 25 are poles of the same (35°) column arc. The pole direction would be the direction from $\mathbf{D}_u$ to 23 25, not 13 17.

The behavior of the arc in Figure 16.3 is typical for contact arcs and, hence, for column arcs. Figure 16.4 is another example, but with more halos. It shows all of the 18° , 20° , and 35° column arcs. Look at the right-hand side of the figure and notice how, in the $\Sigma = 0^\circ$ diagram, the contact direction for each arc is perpendicular to the pole direction of the arc. Then in the $\Sigma = 20^\circ$ and $\Sigma = 40^\circ$ diagrams note how, with increasing sun elevation, each contact point is moving in the direction away from the pole direction. The progress of the 20° and 35° contact points is slower than that of the 18° contact points, since their poles are farther from $\mathbf{D}_u$ (larger s). Also notice that while all of the halos are tending to be circular with increasing sun elevation, the 18° arcs are doing so more quickly, again due to the proximity of their poles to $\mathbf{D}_u$.

The simulations in Figure 16.4 were made with crystals having only pyramid faces—no prism or basal faces. The 9° , 22° , 23° , 24° , and 46° arcs must all be

FIGURE 16.3 (Left and above) Evolution of a contact arc with increasing sun elevation. The pole of the arc is P_u . Its direction (the arrow in the pole diagram, above) from D_u and its angular distance s from D_u are critical. For each sun elevation Σ between 0° and s there are exactly two contact points with the circular halo. Each contact direction (arrows in the halo diagrams, left) makes an angle $\Delta\tau$ with the pole direction. As Σ increases from 0° to s , the angle $\Delta\tau$ increases from 90° to 180° , so that the contact points move in the direction opposite to the pole direction, finally merging to form a single contact point when $\Sigma = s$. For $\Sigma > s$, the arc is detached from the circular halo. The halo shown here is the 18° column arc with wedge 13 25, for which $s = 59^\circ$. The halo and its pole also appear in Figure 16.4.

FIGURE 16.4 (Facing page, left) The 18° , 20° , and 35° column arcs for sun elevations $\Sigma = 0^\circ$, 20° , 40° . There are two 18° column arcs, one 20° column arc, and two 35° column arcs. The 18° column arcs are left-right reflections of each other, as are the 35° column arcs. The 20° column arc is itself left-right symmetric. The poles of the five arcs are shown at the bottom left, where they are labeled with their wedges and $\Delta_{\min}$ -values. (Right) Three of the five arcs shown in the left-hand simulations, together with their contact points (solid dots). The contact points change with sun elevation as explained in Figure 16.3. The poles of the three arcs are shown at the bottom right.

absent, since their ray paths involve prism or basal faces (Table 8.1). Also absent are all arcs that are not refraction arcs; they have been removed artificially in order to clean up the simulations. That leaves just the 18° , 20° , and 35° arcs. By referring to the $\Sigma = 20^\circ$ simulation, you should be able to pick out the 18° , 20° , and 35° arcs in Figure 16.2.

Figure 16.5 is another example to help in sorting out the halos in Figure 16.2. The simulations in this new figure were made with crystals having pyramid faces and fairly long prism faces, but no basal faces. Among other things, the long prism faces strengthen the 9° and 24° arcs, and they weaken the 18° arcs. However, in this simulation even the relatively weak 18° arcs that these crystals would have made have been removed artificially, along with the 22° arcs and all non-refraction arcs, all this being done in the interests of making the remaining

FIGURE 16.5 (Left) Similar to the left-hand diagrams of Figure 16.4 but showing the 9° , 20° , 24° , and 35° column arcs for sun elevation $\Sigma = 20^\circ$. There are two 9° arcs, one 20° arc, four 24° arcs, and two 35° arcs, with the 20° and 35° arcs being the same as in Figure 16.4. (Right) Five of the nine arcs shown in the left-hand simulation, together with their contact points.

halos more intelligible. The 9° and 24° arcs in this figure can be used to identify the 9° and 24° arcs in Figure 16.2.

You should now be able to identify nearly all of the halos in Figure 16.2. You can check your identifications by referring to Figure 16.6, which is the same as Figure 16.2 but with the refraction arcs labeled. In addition to the 9° , 18° , 20° , 24° , and 35° arcs already discussed, there are 22° and 23° arcs. But the 22° arc (the ordinary tangent arc) is inconspicuous here, due to the short prism faces of the crystals. And the 23° labels in the figure are not very helpful in picking out the 23° arc, since, although they correctly indicate the contact points, the arc is indistinguishable from the much stronger 18° arcs near the contact points. The 23° arc does appear at the very bottom of the figure as a highly suncave curve extending upward towards its contact points. This arc is the 23° arc 1 23; the

FIGURE 16.6 Same as Figure 16.2 but with the refraction arcs labeled just sunward of their contact points. The contact points themselves are not indicated, but see also Figures 16.4 and 16.5. The faint parhelic circle and the faint loop passing through the sun are not refraction arcs. $\Sigma = 20^\circ$.

other 23° column arc, 13 2, fails to make contact with its circular halo at this sun elevation and is not labeled. It is so weak here as to be lost among the other halos.

The simulations in Figures 16.2–16.6 are idealized in the extreme. In making them, we did everything possible to make the halos sharply defined and easy to distinguish: the tilts of the crystals were nearly zero, the sun was a point sun, the light was monochromatic, and no diffraction was included. Although we think these simulations are beautiful and pedagogically useful, they are far from realistic.

Some real displays

Figure 16.7 shows a display with odd radius column arcs. It is obvious that some of the halos are odd radius, but how do we know that they are column arcs? The clearest sign is that the apparent 9° halo is elongated in the vertical direction and enhanced at the sides; this halo is not the 9° circular halo but rather the 9° column arcs. The 24° column arcs are present as well, but they are harder to recognize as column arcs, since they look like pieces of the 24° circular halo.

What about some of the other halos in the photo? The truly circular halo is the 22° halo, and the bright arc at the top of the 22° halo is the 22° column arc, that is, the ordinary tangent arc. The 24° column arcs presumably extend into this same region just above the 22° halo, making it difficult to distinguish the various arcs there.

The simulation is a good match for the photo, but it is not perfect. Weak 18° arcs are present in the photo near the two orange dots, but they do not appear in the simulation. On the other hand, weak 20° column arcs are present in the simulation at the left and right just inside the 22° halo, but they do not show up in the photograph.

The colored dots in the photograph are located at the theoretically calculated contact points of the 9° , 18° , 22° , and 24° column arcs. The various arcs look just right near these points, except perhaps at the two uppermost contact points of the 24° arc (purple). In that region the situation is complicated by the brightness of the 22° column arc. No contact points have been indicated in the lower part of the photo. The display is weak there because the halo-making cloud does not extend that far.

Figure 16.8 shows one stage of a Boulder, Colorado, display that was photographed and studied by Paul Neiman [50]. As in Figure 16.7, and indeed as in most displays of odd radius column arcs, it is only the 9° arc that immediately gives away the display as containing odd radius column arcs; here the 9° arc is clearly distorted

FIGURE 16.7 Column arcs and 22° halo. The colored dots are located at the theoretically calculated contact points of the 9°, 18°, 22°, and 24° column arcs. The simulation was made using crystals like the one shown. The crystals were given column orientations as depicted on the sphere and as explained in Figure 5.3. The dashed circle represents the 22° halo. The vignetting of the sun-blocking stick in the photo is due to rather heavy digital unsharp masking. Fairbanks, June 15, 2002. $\Sigma = 41^\circ$.

FIGURE 16.8 Halo display with 9° , 22° , and 24° column arcs. Fainter halos are present as well. Boulder, Colorado, July 21, 1986. The sun elevation is thought to be about 56° . Photo © Paul J. Neiman.

FIGURE 16.9 Halo display with 9° column arcs, closest to the sun. The display is dominated by the beautiful upper tangent arc (22° column arc). Bend, Oregon, June 30, 1988. Photo © O. Richard Norton, Science Graphics.

in the vertical direction rather than being truly circular. Further out from the sun, at the 10:30 and (to a lesser extent) 1:30 positions, we are probably seeing bright 24° column arcs, but the identification is complicated by the proximity of the 22° column arc.

Figure 16.9 is from a fine display photographed by O. Richard Norton in Oregon. Again the characteristic shape of the 9° column arcs is clear. Initially we were puzzled by the absence of the 24° column arcs in this display; we expected them to accompany the rather strong 9° column arcs. However, we found it easy to make a simulation in which the 24° arcs were so much weaker than the 9° arcs as to be nearly invisible. No exotic crystal shapes or orientations were needed.

Figure 16.10 shows the best display of odd radius column arcs that we know of. However, the photograph is a composite of five photos taken several minutes apart, and there was never a moment when the display itself was as good as in the composite. But presumably if the halo-making cloud had been more extensive, the display would have looked like that in the composite. The 9° , 18° , 20° , 22° , 24° , and 35° column arcs were all present.

The simulation in the same figure shows the expected column arcs for the relevant sun elevation. The simulation is not perfect but, overall, it is not a bad match for the photo. There is an arc near the top of the photo that is missing in the simulation, but it is probably the upper 23° plate arc and, not being a column arc, would not be expected to appear in the simulation. The halos at the bottom of the simulation are outside the region of the halo-making cloud and hence do not appear in the photo.

This display is discussed in more detail by Riikonen et al [59]. They also analyze another display of odd radius column arcs, a display that had been considered much earlier—in 1975—by Hattinga Verschure [24]. Hattinga Verschure had recognized the strangeness of the halos and had reasoned correctly that they were odd radius column arcs. We should also mention Tricker [79], who in 1979 attempted to calculate the theoretical shapes of various odd radius column arcs. Neither the Hattinga Verschure nor the Tricker articles got the response that they probably deserved; they evidently did not motivate people to watch for odd radius column arcs.

As you look over our simulations and photographs of odd radius column arcs, it will be obvious that in this chapter we let theory get ahead of observation. With the exception of Figure 16.10, the existing photographs of odd radius column arcs are not remotely comparable to the idealized simulations in Figures 16.2–16.6.

FIGURE 16.10 Odd radius column arcs, Oulu, Finland, September 17, 2001. The photo is a composite of five photos taken several minutes apart. The heavy black dots in the simulation are theoretical contact points of the 9°, 18°, 20°, 22°, 24° and 35° column arcs. No contact points are shown in the lower part of the simulation, since the display itself did not extend into that region. The crystals used in making the simulation had shapes and tilts as indicated. $\Sigma = 27^\circ$. Photo © Marko Riikonen.

Although we doubt that we will ever see odd radius column arcs as fine as the ones in those simulations, we know that we—or perhaps you—can do better than our current photos. We know that odd radius column arcs exist, and we know that the halo community is still largely oblivious to them. It takes an informed observer to be able to recognize that an odd radius column arc is something out of the ordinary and thus worth photographing. So we think that with increased awareness better photos will be forthcoming. Then perhaps the simulations will not seem so idealized.

Odd Radius Parry Arcs

No odd radius Parry arcs have ever been reported. But from studying halo displays like the one in Figures 6.7 and 6.8 we know that prismatic crystals can sometimes take on Parry orientations as they fall. If those same crystals had small pyramidal faces in addition to their prism and basal faces, wouldn't they still take on Parry orientations? If so, then odd radius Parry arcs should be possible, and eventually someone will see one. We touch on odd radius Parry arcs here mainly as an inducement to watch for them. But seeing one is much more of a long shot than seeing a good display of odd radius column arcs. People with the best chances of seeing an odd radius Parry arc are those lucky few who are able to spend extended periods in the Antarctic interior, where ordinary Parry arcs are common. But even for those folks it will take extraordinary perseverance, or luck.

The simulation in Figure 17.1 shows odd radius Parry arcs for a sun elevation of 20°. The crystal orientations were barely allowed to deviate from ideal Parry orientations, so the simulation shows more than one could normally hope for. Probably the arcs most likely ever to be seen are the four strong arcs closest to the sun at the left and right. They are the 9° Parry arcs with ray paths 8 25, 28 5, 4 27, and 24 7. Their locations in the sky are consistent with their poles (Figure 17.2). That is, since Parry arcs are non-contact arcs, the direction of each arc from the sun is about the same as the direction of its pole from $\mathbf{D}_u$.

According to the pole diagram, there are two more 9° Parry arcs, namely the upper and lower symmetric arcs 3 26 and 23 6. But the lower symmetric arc has its pole far too close to $\mathbf{D}_u$ to be visible at the relatively low sun of this simulation. And although the upper symmetric arc is visible in the simulation—it is the suncave arc above and nearest the sun—its pole is also fairly close to $\mathbf{D}_u$, so that

FIGURE 17.1 Odd radius Parry arcs. The simulation was made using Parry-oriented pyramidal crystals shaped like the one in Figure 17.2. $\Sigma = 20^\circ$.

FIGURE 17.2 (Left) Parry-oriented pyramidal crystal. Faces 3 and 6 are horizontal. (Right) Poles of the six 9° Parry arcs.

at this sun elevation the arc is substantially more than 9° from the sun and would be weak and diffuse, were not the tilts of the crystals so small.

Many odd radius Parry arcs are theoretically possible. The pole diagram of Figure 17.2 shows the possibilities only for $\Delta_{\min} = 9^\circ$. There is an analogous pole diagram for each of $\Delta_{\min} = 9^\circ, 18^\circ, 20^\circ, \dots$

Sometimes I (Tape) think that contemplating odd radius Parry arcs is pure fantasy, and that nobody will ever see one. Then I am reminded of a halo simulation that

I made after my first season at the South Pole. It was a simulation of halos from Parry-oriented prismatic crystals. (It had nothing to do with odd radius halos.) That first season I had seen some pretty good 22° Parry arcs, so—mostly for amusement—I turned the crystal tilts in the simulation way down and I turned the numbers of crystals way up. The resulting simulation was exquisite, but it seemed farfetched; I had never seen anything like it. Yet it was not many weeks into the second season at Pole that a display appeared that was nearly the equal of the simulation. (It was not quite as good as the display in Figures 6.7 and 6.8.) So the point is that it is sometimes hard to predict what is possible. We have spent a lot of time watching for odd radius Parry arcs, and we have seen nothing. But maybe one of you will do better.

Other Wedge Angles?

So far in this book we have studied halos made by ice crystals having prism, basal, and $\{10\bar{1}1\}$ pyramid faces. These crystals have the familiar wedge angles listed in Table 8.1. Could there be other wedge angles on ice crystals? If so, then there might be other halos as well. What (contemporary) evidence is there for the existence of new wedge angles?

The most obvious way of getting new wedge angles is to have new crystallographic faces, that is, faces other than prism, basal, and $\{10\bar{1}1\}$ pyramid faces. In looking at countless atmospheric crystals, we have not seen evidence for such faces. However, there are many complex or small crystals where it is difficult to identify the faces crystallographically. Moreover, we do not feel that in our observations we have yet exhausted the variety of crystals. New crystallographic faces may yet turn up.

In any case, new wedge angles do not require new crystallographic faces. Figure 18.1 is an example. It shows a polycrystal, that is, a “crystal” made up of two or more component crystals. Any wedge whose two faces are in different components is apt to have a novel wedge angle.

Polycrystals are common in some crystal samples, but they are relevant to halo theory only if the component crystals are joined to each other in some systematic way from one polycrystal to the next, so that the same novel wedge angle can occur in many crystals and thus perhaps make a halo. For polycrystals similar to that in Figure 18.1, in which the components of the polycrystal are more or less equidimensional pyramidal crystals, we have so far recognized no underlying crystallographic principle that controls the joining of the components; rather the components seem to be joined randomly. We therefore do not expect to get new halos from these crystals.

FIGURE 18.1 Polycrystal whose components are pyramidal crystals. Photo © Kenneth P. Severin.

FIGURE 18.2 Crossed pyramidal plate crystal, a candidate for novel wedge angles. Photo © Kenneth P. Severin.

The polycrystal in Figure 18.2 is another story. It consists of three crossed pyramidal plates joined in a very definite way. In fact, the line of intersection of any two of the plates is parallel to an a -axis in each, and this implies that the orientations of the plates with respect to one another are fixed.¹ Crystals like these would make new halos if they were large enough and occurred in sufficient numbers.

Thus the ice crystals themselves suggest the possibility of wedge angles other than those that make the standard odd radius halos that we have considered so far. Halo displays also give strong evidence for new wedge angles, as we will see momentarily.

M-arc

Given that the modern theory of halos is several centuries old, it may come as a surprise that there are halos still unexplained. Perhaps even more surprising is the fact that new halos are still being reported from time to time. The M-arc is such a halo. The oldest record of it that we are aware of consists of a photograph by Jon Nickles in Anchorage, Alaska, in 1978 and published in *Alaska Geographic* [1, page 54]. The significance of the photo was not recognized until later, after Moilanen [47] had called attention to the existence of the new halo in displays from Finland and elsewhere.

In the photos in Figure 18.3 the M-arc is the vee-shaped arc above the sun and well inside the 22° halo. It might be mistaken for an upper 9° plate arc, but

¹ Kobayashi and Kuroda [36] offer some explanations for this configuration.

FIGURE 18.3 M-arc, the vee-shaped arc directly above the sun. (*Top*) Viitasaari, Finland, January 8, 1999. Photo © Rainer Vilkkilä. (*Bottom*) Anchorage, Alaska. Photo © Evelyn Trabant.

neither the shape nor location is quite right. Moreover, a 9° plate arc as bright and sharply defined as these M-arcs would be expected to be accompanied by other odd radius plate arcs, which we do not see in the photos.

The M-arc can easily be simulated using a wedge having wedge angle $\alpha \approx 34^\circ$, but what real ice crystals have such a wedge angle? Crossed plate crystals do, and so do various other polycrystals. Simulations made using these crystals can passably reproduce the M-arc, but they tend to produce other halos as well, halos that are not seen in the real displays. The M-arc remains an open problem.

Whatever its explanation, the M-arc seems to be an example of a “doubly odd radius” halo, that is, a halo that cannot be produced by our standard Steinmetz and Weickmann pyramidal crystal model (Figure 9.5).

The Lascar display

The remarkable Lascar halo display was seen by Marko Riikonen, Leena Virta, and Daniel Sullivan while camped on the flanks of Lascar volcano in northern Chile in 1997. Both Riikonen and Virta were members of the Finnish Halo Observers Network, and they had the halo expertise to appreciate what they were seeing. They managed to take approximately 100 photographs of the display over the course of two days. One of their photos is reproduced in Figure 18.4.

Without describing the display in detail, we will just say that it seems to contain at least one doubly odd radius halo. In fact, one after another of the Riikonen–Virta photos shows an arc with $\Delta_{\min}$ somewhere around 28° —nowhere near the standard $\Delta_{\min}$ -values of Table 8.1. In Figure 18.4 this arc appears faintly in the 12:00 position at the level of the two yellow arrows. Suggestions of a 28° circular halo are also present, especially to the left and right of the sun.

Riikonen et al [62] analyzed the Lascar display and concluded that many of the halos were odd radius plate arcs from standard pyramidal crystals, as in our Chapter 15, but that other halos, in particular the 28° arc, were due to a rare variety of ice known as cubic ice. Cubic ice has internal symmetry that is cubic (isometric) rather than hexagonal, and cubic ice might therefore come in cubeoctahedral crystals, that is, crystals having cubic and octahedral faces. It was these cubic and octahedral faces that gave Riikonen and his colleagues the wedge angles that they needed to reproduce the strange halo radii in the Lascar display. Using cubeoctahedral crystals together with standard pyramidal crystals, they were able to make simulations that approximated the Lascar photographs.

We—Tape and Moilanen—disagree with each other somewhat on the explanation of the Lascar display. Moilanen, who was one of Riikonen’s coauthors, believes that the cubic ice explanation may yet turn out to be correct, though he sees some weaknesses in it. Tape is nervous about the cubic ice explanation for

FIGURE 18.4 Strongly enhanced photo of the Lascar halo display. Riikonen et al [62] believe the bright arcs here to be 18° and 23° plate arcs. The faint arc between the arrows is a “doubly odd radius” arc. Here it almost looks like an artifact of the enhancement, but it also appears clearly in the original slide. November 27, 1997. Photo © Marko Riikonen.

several reasons, but he does not have much in the way of concrete alternatives to offer. He is haunted by the seemingly outlandish possibility that the halo-making crystals might not be ice at all. In any case, regardless of who is right, the explanation of the Lascar display seems to require something other than our standard pyramidal crystal.

In passing we mention Scheiner’s halo, a circular halo that is supposed to have a radius of 28° . Well before the Lascar display, Whalley [89] had suggested that cubic ice might be responsible for Scheiner’s halo. Whalley’s article lists half a dozen reported sightings of the halo, including the original sighting by the astronomer Christopher Scheiner at Rome in 1629.

We find these reports to be unconvincing. All of them are old. Several can easily be dismissed as sightings of incomplete tangent arcs. Scheiner’s observation itself seems to be lost, and what is usually cited instead is a description and drawing by Gassendi [19], with whom Scheiner had corresponded. Until the Lascar display, we ourselves did not believe in a 28° halo, and we are still not convinced that the Lascar display has any relation to these old and suspect reports.

FIGURE 18.5 Puzzling halo display, South Pole, December 10-11, 1998. The brightest halo is a 20° halo. Nearest the sun in the upper photo is a five or six degree halo (arrow), then next a 9° halo. The colored dots in the lower photo are at the indicated angular distances from the sun (the first six $\Delta_{\min}$ -values from Table 8.1). The dots are reference points only; there may or may not be a halo nearby. The upper photo was taken with a 15 mm lens, the lower with a 20 mm. Both are strongly enhanced with digital unsharp masking. The apparent halo about 28° above the sun is largely an artifact of the enhancement. $\Sigma=23^\circ$.

5° Halo

In a murky overcast sky at the South Pole in December of 1998, we watched an odd radius display evolve over the course of a couple of hours. We photographed the halos, and we collected and photographed low level atmospheric crystals. However, due to the overcast conditions and the resulting absence of sparkles, we could not be sure that the halos were originating in low level crystals, and we therefore could not be sure that the crystals that we collected were representative of those making the halos. Some of the halos are shown in Figures 18.5 and 18.7, and some of the crystals are shown in Figures 10.8 and 18.6.

At the time of the display we thought we knew what we were seeing, but upon returning home and developing the film, one of us—Moilanen—noticed in many of the slides a 5° halo and perhaps a 12° halo, neither of which we had recognized earlier. These halos would of course be further examples of doubly odd radius halos. The 5° halo is quite clear in many of the original slides, being thin and sharply defined, and we hope that it will survive reproduction in Figure 18.5. The 12° halo is much fainter and more diffuse, if indeed it exists at all. Perhaps Figure 18.7 has a suggestion of it?

More surprises were yet to come. We had assumed that the brightest and most conspicuous circular halo in the display was the 18° halo, but measurements of the halo radius from the photos proved otherwise. This is clear from the lower photo in Figure 18.5, where the colored dots are at the indicated angular distances from the sun. The sunward edge of the halo in question is nearly at the yellow dots, which are about 20°, not 18°, from the sun. (Our 15 mm lens is not calibrated for

FIGURE 18.6 Some crystals that fell during the halo display of Figure 18.5.

FIGURE 18.7 A later stage of the display shown in Figure 18.5. The lower 9° plate arc, the 22° plate arcs, the upper 23° plate arc, and the infralateral 24° plate arcs seem clear. The circular halos, except for the 9° halo, are difficult to identify. The photo is strongly enhanced.

making angular measurements, which is why colored dots were not put on the upper photo.)

Using only our standard pyramidal crystal, we do not see how to make a 20° halo without making either a strong 18° or a strong 22° halo. But there is no 18° halo present in the display, and we are not convinced that the 22° halo is there either—if it is, it is not strong. So even without the 5° and 12° halos, this display is a mystery.

The crystal photos may well contain the key to the display, but the crystals are complex, and so far we have not been able to decipher them. Perhaps there is some sort of twinning going on.

Figure 18.7 shows a later stage of the same display. By that time, what appear to be standard odd radius plate arcs had become conspicuous. Some, at least, of the responsible crystals seem therefore to have been ordinary pyramids. But the circular halos are still hard to identify with any certainty, other than perhaps the 9° halo.

Parheliion flares

In the halo display shown in Figure 18.8 a tall column of light, bending ever so slightly outward, seems to emanate from each parheliion. We do not understand

FIGURE 18.8 Parhelion “flares”—the two tall columns of light seemingly emanating from the parhelia. Odd radius arcs are present as well. The origin of the parhelion flares is not known. Fairbanks, December 30, 2002.

these halos. Until we do, we propose to call them parhelion “flares.” Parhelion flares as tall and bright as these are rare, but we fairly often see rudimentary and shorter parhelion flares in low level displays in Fairbanks. They tend to be accompanied by unusually tall and bright sun pillars, as here.

Parhelion flares may or may not turn out to involve novel wedge angles; we just do not know. And they may or may not be related to the parhelia.

Elliptical halos

On rare occasions one or more small rings, vertically elongated and often incomplete, appear around the sun or moon (Figure 18.9). These “elliptical halos” can be passably simulated using thin pyramidal crystals with angle x very large—something more than 80° , depending on the size of the halo. The crystals would thus be something like the pyramidal crystal of Galle in Figure 11.1, but thinner, with the pyramid faces sloping much more gently and with the prism faces being smaller or absent.

A glance at Table E.2 shows that pyramid faces with large x are completely unreasonable from a crystallographic point of view; in the entire table the largest entry for x is only 72.6° . The $\{1\ 0\ \bar{1}\ 11\}$ pyramid faces would give $x = 80^\circ$, from Eq. (9.6), but, again, these faces are ridiculous crystallographically. So we do not know how elliptical halos form.

A recent article by Sillanpää et al [68] contains much more information on elliptical halos, including an analysis of the display in Figure 18.9, as well as references to other work. The article also discusses Bottlinger's rings, a phenomenon resembling elliptical halos but surrounding the subsun rather than the sun.

FIGURE 18.9 Elliptical halo display, Kokkola, Finland, February 13, 1997. Photo © Martti Penttinen.

Hevel's halo

Hevel's halo is supposed to be a circular halo of radius 90° . It has been treated by many authors, including one of us [76], and we have all come up empty. That is, nobody understands how such a halo could occur. We have little to add here, so we will be brief.

The halo was reported by Johannes Hevelius in his famous Danzig display of 1661. His original Latin description is in the rare *Mercurius in Sole visus Gedani* ... [26], but most of it can also be found in Smith's *Opticks* [69], in English translation.² Hevelius was quite explicit about the size and shape of the halo; it was circular, though incomplete, and its radius was 90° . Hevelius was one of the premier astronomers of his time and was superbly equipped to make observations of angular distances in the sky, so it is hard to believe that he got the description wrong by much.

² The *Opticks* version has some weaknesses, the most serious being the omission of Hevelius' explicit statement that the halo in question crossed the ecliptic at right angles, as a 90° sun-centered halo must. If Hevelius was correct on this point, then the halo in question could not have been the subhelic arc. A French translation of Hevelius' description of the display, as well as his original Latin description, can be found in the complete works of Huygens [33, pp 424–429].

Hevel's halo is too big to be a circular halo in the conventional sense of Chapter 8. A halo whose ray path enters and then directly exits the crystal, as we assumed there, can have radius no larger than 80.5° (page 77). So if Hevel's halo is a truly circular 90° halo, then some new mechanism is needed for its explanation. No such mechanism has been forthcoming.

In 1980 Greenler [21] suggested that perhaps Hevelius was not seeing a circular halo at all, but was seeing the subhelic arc instead. At first, Greenler's suggestion seems unlikely, almost preposterous. The subhelic arc is just not close to being a 90° circular halo, and Hevelius, as we said, should have been too good an observer to confuse the two. But is the alternative any more likely? The alternative is that Hevelius in 1661 saw a halo that has been neither seen nor explained since.

If we absolutely had to bet, we would probably go with the subhelic arc hypothesis. One objection to it has recently been removed, namely, that a subhelic arc should have been accompanied by a Wegener arc, and that Hevelius did not see any Wegener arc. Recent halo displays like that in Figure 5.9 have shown that the subhelic arc can occur without the Wegener arc. But the removal of this objection does not change things much. Until somebody gets a photograph of a 90° halo, Hevel's halo will continue to fuel late night pub conversations at halo conferences.

44/46° parhelia

The orange arrow in the photograph in Figure 18.10 points to a halo that might easily be misidentified as a “46° parhelion,” were it not for presence of the 46° halo.³ But the halo in question is clearly closer to the sun than is the 46° halo; it cannot be a true 46° parhelion.

We believe the halo is a secondary halo, a parhelion of a parhelion. That is, the ordinary 22° parhelion is acting as a light source and creating its own 22° parhelia. One of them would be at the sun and would of course be overpowered by the sun. The other is about 44° from the sun and is the halo of interest here. The blinding intensity of the ordinary 22° parhelion in the photo is just what we would expect, if the parhelion is to be able to make its own halos.

The idea of secondary halos goes back at least as far as Bravais [9] in 1847. Secondary halos were added to halo simulations by Tränkle and Greenler [78] in 1987. We mention secondary halos here mostly to caution that an odd radius does not automatically imply an odd wedge angle. In the photo we seem to have a halo with $\Delta_{\min} \approx 44^\circ$, but its explanation does not lie in a wedge angle $\alpha \approx 89^\circ$, as would follow from Eq. (8.1).

³ One can argue about whether the apparent 46° halo here might in fact be the supralateral arc, but it does not matter much, since for this sun elevation the two halos would nearly coincide in this part of the sky.

FIGURE 18.10 Bright 22° parheliion at left, 44° parheliion (orange arrow), and 46° halo (green arrow). The sun is out of the photo at the left. Teuva, Finland, November 21, 2004. Photo © Heikki Mahlamäki.

It is a mistake to think that because halo theory is old, it is also complete. There are halos, like the parheliion flares, whose existence is indisputable but whose origin is obscure. Moreover, there will be new halos, new in the sense that they are being recognized for the first time. Some of them will have been anticipated, as would be the 9° Parry arcs if they were to turn up. Others will be completely new, as was the M-arc when it was first seen. The fun is not over.

Pattern for Crystal Model

With a crystal model in your hands you can find halo poles easily (Chapter 14). The pattern below is for a crystal with $\{10\bar{1}1\}$ pyramid faces and $c/a = 1.63$ as usual. We suggest enlarging the pattern on a photocopier and then constructing the model from cardboard.

TABLE B.1 Correspondence between some traditional halo names and names derived from the halo classification of Chapters 12–14.

Traditional name	Name from halo classification
Parhelia	22° plate arcs
Tangent arc	22° column arc
Infralateral and supralateral arcs	46° column arcs
Circumzenith arc	Upper symmetric 46° plate (or Parry) arc
Circumhorizon arc	Lower symmetric 46° plate (or Parry) arc
Parry infralateral arcs	Infralateral (46°) Parry arcs
Parry supralateral arcs	Supralateral (46°) Parry arcs

Halo Terminology

Halo names arose over time. Some halos, such as the Wegener arc, bear the name of an observer or halo scientist. Others, such as the circumzenith arc, are named according to their location or appearance. When you begin to consider odd radius halos, you soon realize that some sort of more uniform and more conceptual terminology is desirable. Only the most devoted enthusiasts will be able to remember that the halo of van Buijsen, say, is the 9° halo, whereas the halo of Feuillée is the 35° halo.

In this book we therefore introduced a uniform nomenclature scheme for refraction halos, its basis being the halo classification described in Chapters 12–14. In some cases the new scheme resulted in new names for familiar halos. We used the new names when we felt that they added clarity.

We are not, however, suggesting a massive renaming of old halos. Most of the time the circumzenith arc, for example, will continue to be the circumzenith arc rather than the upper symmetric 46° plate (or Parry) arc. But there are times when we get some insight into the circumzenith arc by realizing that it is indeed the upper symmetric 46° plate arc. And surely we understand the upper symmetric 23° plate arc better by recognizing its close relation with the circumzenith arc—a relation that is emphasized by the new nomenclature.

A disadvantage of the new nomenclature is of course that some refraction halos now have more than one name. They are listed in Table B.1.

For the arcs often called the heliac arc and Tricker arc we have instead used the names helioc arc and anthelic arc, respectively, in order to stress the tight relation among the helioc, anthelic, subhelic, and subanthelic arcs. These two changes do not appear in the table, since they do not pertain to refraction halos.

Halo Observation and Photography

Anyone who sees a rare halo can obtain useful photographs. In the case of odd radius halos, the photographs will be more valuable scientifically if angular measurements can be made from the photos, so that halo radii can be measured. In this appendix we offer some general suggestions for halo photography and then some specific precautions that need to be taken so that angular distances can be found from the photos.

When a good halo display comes along, there are no real tricks to getting good photographs of it. Just block the sun with something (but see below) and fire away. Do not stint on film; the display is apt to change from moment to moment, and there are no second chances. Bracket your exposures, especially for wide angle lenses. For lunar displays use a tripod, of course, and try exposures from 5 to 40 seconds with ASA400 film and with the aperture wide open. With a digital camera you can check your photos as you take them, thus reducing the number of different exposure times, but we still recommend some bracketing.

Although there are no major secrets to halo photography, there are of course things that can go wrong. Probably the best advice we can give is to experiment. Don't wait for the big display to test your operation. Get the bugs out by photographing a few mediocre displays that you do not care about.

Your first attempt at halo photography will reveal the huge size of halos and the consequent need for wide angle lenses. A 28mm lens has a wide enough field of view to include both parhelia. A 20mm lens is enough for the entire 22° halo, and a 15mm lens is enough for the supralateral arc. (The preceding numbers are for traditional film cameras; their equivalents for a digital camera can normally be found in the camera manual.) Zoom lenses are not so good if angular measurements are ever to be made from the photographs.

One recommendation specific to digital: Archive the original files and do not tamper with them. Any processing should be done on copies. It is easy to overdo digital enhancement, the result being an ugly photo that may contain artifacts mimicking halos.

While we are on the subject of digital: Digital cameras are vulnerable to cold and condensation. If you anticipate photographing low level halo displays, especially lunar displays, where your camera will be exposed to prolonged cold, an old-fashioned mechanical film camera may be preferable to digital.

What about measuring halo radii from a photo? The ideal halo display for doing so is a lunar display in which stars as well as halos are visible (e.g., Figure 15.4). The main thing then is to **record the time and place of each photo**, so that celestial coordinates of the moon can be found. (If your camera is digital, the time will be included in the EXIF data, which of course should be saved.) The angular distance from the moon to any star can then be calculated. If there are stars in your photo near the halos, you will be able to make some estimates of the halo radii.

But good lunar halo displays are not common, especially ones with lots of stars. So normally you must resort to something like the star triangle method of Appendix D. The method requires some effort, but it will work for any photo, daytime or nighttime. For the photographer the main thing to remember is that **the disk of the sun (or moon) should be clear in the photo**. This means having two pieces of polarizing film available, so that they can be crossed and interposed between the camera and the sun, as in Figure 15.2. Polarizing film is available through many scientific supply companies.

Recently Marko Riikonen has been experimenting with a technique that superposes large numbers of photos of the same halo display. We suspect this is the wave of the future in halo photography. The technique smoothes out inhomogeneities in the clouds so that the halos stand out better against the background, and it reduces any intensity variations in the halos that result from variability in the clouds. The result is a composite image that shows far more than any of its component images and indeed shows far more than would be seen by an observer during the display. Figure C.1 is one of Riikonen's composites. He stresses that the display itself was not outstanding.

Miscellaneous observing suggestions If you have come this far in the book, you will know something about halo observing. We nevertheless remind you to look at the entire sky. That is, do not be completely seduced by the sunward side of the sky; look overhead as well, and look toward the anthelic region. Look at the whole sky.

Some halos are sufficiently polarized so that the polarization can be detected by observing the halos through polarizing film or polarized sunglasses. Try it first

FIGURE C.1 Composite of many images of the same odd radius halo display. Fifty-one photographs, taken over an interval of ten minutes, were enhanced with digital unsharp masking and then superposed to make the composite. The composite shows much more than was visible at the time of the display. Kontiolahti, Finland, August 20, 2005. Photo © Marko Riikonen.

with a bright parheliion. Just observe the parheliion through the polarized glasses and then rotate the glasses ninety degrees. You will see a small horizontal shift in the inner edge of the parheliion.

If you own a digital camera with a polarizing filter, you can try a fancier version of the previous experiment. With the camera mounted on a tripod, zoom in on the parheliion and photograph it first with the polarizer positioned so that the parheliion is closest to the sun, then again so that the parheliion is farthest from the sun. The resulting two photos can then be superimposed and animated to show the shift. (The disadvantage of the traditional film camera here is that aligning the photos can be an ordeal.)

The shift in a halo due to the polarization tells something about the halo, and it can even be decisive in identifying certain odd radius halos. So if you are lucky enough to see a bright odd radius display, try to remember—in the excitement of

the moment—to look for the shift in each halo. If you are able to detect the shift, note in writing which way it goes, and note the corresponding orientation of your polarized glasses. For much more on the role of polarization in halos, including many observational tips, see the article by Können [40]. Können apparently was the first to notice the shift.

If you live in a cold place, you can easily make plastic replicas of atmospheric ice crystals during low level halo displays, and this information, too, can be useful if the halos are exotic. All you need are some glass slides and acrylic spray. We use Krylon clear acrylic spray, which is carried by many hardware and paint stores. When crystals are falling, just spray a slide with the Krylon and then wave it through the air for twenty seconds or so to intercept the crystals. (Both the slide and the Krylon should be cooled before spraying, but the Krylon should not be cooled so much that it comes out of the can in a trickle rather than a spray.) The sprayed slide should be left outside in the cold for an hour to allow the crystals to evaporate. Then the slide can be brought inside and examined with a microscope or strong hand lens. If the halo display is important, we generally make several replica slides, varying the amount of spray from one to the next. Usually the replicas are fairly good, but not always. They are never the equal of photographs of the crystals themselves, but cold weather photography with a microscope is a more demanding project.

At the time of this writing there were organized networks of halo observers in Finland, Germany, Holland, and the Czech Republic. Such networks, with many alert observers spread over large areas, have had a huge impact on halo studies, by providing photographs of rare halo displays that would never have been seen without the networks. These networks welcome new members.

Finally, some obvious but crucial reminders: Watch the sky, and keep the camera, the polarizers, and the replica materials with you.

From Pixels to Degrees

Accurate angular measurements of halo radii are central to our story, but getting them turns out to be a challenge. In this appendix we explain how to measure angular distances in a halo photograph, partly so that people can do it for themselves, but mainly to give some confidence in our measurements.

Here we are not concerned with the problem of pinpointing the (always fuzzy) edge of a halo. Instead we imagine that two points on a halo photo have already been specified. The problem is then to find the angular distance between them.

Getting angular distances from a photograph is more complicated than you may imagine. Whereas on a map of New York City, say, you will usually find a scale of miles, you will not find one on a map of North America. That is because on the map of North America the scale of miles changes from point to point. (It changes with direction as well.) A wide angle photograph of the sky is completely analogous to a map of a large region of the earth. There is no single conversion factor from inches (or pixels) on the photo to degrees in the sky. This is seen dramatically in Figure 6.8, where the 22° circular halo appears grossly distorted at the right of the photo. How, then, do we get angular distances from a halo photo?

The star triangle method

The method that we use is the star triangle method. It is based on the fact that celestial coordinates of stars are known—they can be found from astronomical software packages, for example. We superimpose our halo photo on a photo of a star field taken with the same lens as the halo photo. Points in the halo photo can then be treated as if they were points in the star photo. That is, their

“celestial coordinates” can be found from the celestial coordinates of nearby stars. The angular separation between any two points in the halo photo is then easily computed from their celestial coordinates. The actual handling of the photos is best done electronically, that is, by computer; although the method can in principle be carried out physically, in practice it doesn’t work.

Figure D.1 is an example. It shows how to find the angular distance between the sun and the upper right purple dot in the halo photo of Figure 16.7. The halo photo has been superimposed on a suitable star photo—the “background

FIGURE D.1 The star triangle method for finding the angular separation between two points in a photo. Here the points are the white dot (at the sun) and the upper right purple dot in the halo photo of Figure 16.7. The halo photo has been placed electronically on a star field photo, and the opacity of the halo photo has been turned down so that the stars are visible; here the halo photo is barely visible on top of the star photo. The “celestial coordinates” of the white dot are found by interpolation from the celestial coordinates of the three nearby stars 1 Gem, ζ Tau, and 126 Tau. The celestial coordinates of the purple dot are similarly found from the celestial coordinates of α Aur, ϵ Aur, and η Aur. The angular separation of the two dots is then calculated from their celestial coordinates.

photo”—and the opacity of the halo photo has been turned down so that the stars are visible. In the halo photo you can still make out the sun-blocking stick, but not much more. A small “star triangle” has been chosen surrounding the white dot (the sun position); its vertices are the stars 1 Gem, ζ Tau, and 126 Tau, as shown. The pixel coordinates of the three vertex stars and the white dot are found from the photo. These pixel coordinates are then used to calculate the celestial coordinates of the white dot by interpolation from the celestial coordinates of the vertex stars. The celestial coordinates of the purple dot are found the same way, but using the star triangle with vertices α Aur, ϵ Aur, and η Aur. The angular separation between the white and purple dot is then calculated from their celestial coordinates.

We have programmed the computer to do most of the work. Given pixel coordinates for two points in the photo, the computer will find the best star triangle about each point and then proceed as above to find the angular separation between the points. Figure D.2 shows the graphical output of the computer when asked to measure the angular separation between the same white and purple dots as before.

FIGURE D.2 Graphical output of the computer for the angular measurement of Figure D.1. Each black dot is a star in the star field photo. The white dot and the purple dot are the two points whose separation was to be found, the same as in Figure D.1. We had only to measure the pixel coordinates of the two points in the photo, and the computer did the rest: It found the best star triangle (green) for each point, used the star triangles to calculate the celestial coordinates of the two points, and found their angular separation to be 23.81° .

But the computer is not magic, and it needs to be told what stars it has to work with. We must give it pixel coordinates for about 200 stars distributed fairly evenly over the photo.¹ This needs to be done once for each camera lens used in the halo photography. That is, a different background star photo is needed for each lens.

Not just any region of sky will serve as the background photo. Best is a region that is far from the celestial poles and also well above the horizon. Staying away from the poles makes interpolation in star triangles simpler. Staying away from the horizon minimizes distortion from atmospheric refraction. This does not of course eliminate possible distortion in the halo photo due to atmospheric refraction; see page 86.

Checking the star triangle method

How confident can we be in angular separations found by the star triangle method? We believe the method to be accurate to within 0.2° —and usually much better—so long as the points whose separation is to be measured are not too close to the edge of the photo, so long as there are some background stars nearby, and so long as the two photos are carefully aligned.

One way to check the star triangle method is to apply it not to a halo photo but to another star photo. We superimpose the new star photo on the same background star photo as before. The angular separation between two given stars in the new photo can be calculated by using star triangles from the background photo, just as if the new star photo were a halo photo. But the true angular separation between the two stars is of course known, and so the calculated separation can be compared with the true separation.

The upper diagram in Figure D.3 is an example. The black stars are the background stars, the same as in Figure D.1, and the four red stars are from a different star photo. When the black stars are used to compute the angular separations among the four red stars, the six separations are all found to be within 0.13° of their true values; the results are good.

In the lower diagram the photo with the red stars has been offset two millimeters horizontally and one millimeter vertically. (The photos are ordinary 35mm slides.) When the black stars are used as before to calculate the angular separations among the four red stars, some of the resulting separations are now in error by almost two degrees. These errors are far too large if we are trying, say, to identify a halo that could be either the 22° , 23° , or 24° halo. In the star triangle method good alignment of the two photos is crucial.

¹ We could make do with far fewer stars, since we only need background stars in the vicinity of points whose angular separation is to be found. But a large number of stars is preferable, largely for the sake of internal consistency.

FIGURE D.3 (Top) A check of the star triangle method. Instead of a halo photo, another star photo (red stars) was placed on our standard background star photo (black stars). Angular separations among the four red stars were calculated using star triangles from the black stars. The numbers give the errors in the calculated separations—that is, the differences between the calculated values and the true values. (Bottom) Same but with a small misalignment of the two photos. The errors in the angular separations are now far too large to be acceptable.

Alignment is trivial for photos taken with a digital camera. With a film camera, however, alignment can be daunting. Both images must be visible in their entirety, right up to the edge where they meet the unexposed, black part of the film (e.g. Figure D.1). This means that slides cannot be scanned in their mounts. It means that the scanner must not crop the images. It means that the sky in the star photo must be light enough so that the edge of the image can be discerned. And so forth. You can now see why the two photos need to be superposed electronically, rather than, say, by projecting them together on a basement wall, as we did for many years, with dismal results.

Avoiding the star triangle method?

Once we have gone to the trouble of measuring pixel coordinates for a large number of stars in the background photo, it does not take much extra effort to understand how the lens works, that is, to understand how angular distances in the sky are related to linear distances in the photo.

If the lens is aimed at the center of a spoked wheel, the wheel will look right when photographed, with the spokes seemingly undistorted. Thus the action that we are trying to capture is radial. For each star we therefore let r and ϕ be the linear and angular distances, respectively, from the star to the center of the photo. The distance r is measured directly from the photo, while ϕ is calculated from the celestial coordinates of the star and the center of the photo, the celestial coordinates of the center being found by the star triangle method. For the stars α Ori and α Aur, for example, the corresponding r and ϕ values are given in the upper diagram of Figure D.4. Expecting that ϕ will be a function of r , we plot the points (r, ϕ) for all of the stars in the diagram. The result is shown in the lower diagram. The curve $\phi(r)$ through the points is what we are after; it tells how points in the sky are related to points in the photo.

Most camera lenses are what is known as rectilinear; with such lenses, straight lines on an object will appear straight in the photo. For a perfectly rectilinear lens the dependence of ϕ on r is given by what we will call the rectilinear approximation, namely,

$$r = f \tan \phi, \tag{D.1}$$

where f is the focal length of the lens. The star photo depicted in Figure D.4 was made with a 20mm lens. With $f = 20\text{mm}$, the rectilinear approximation [Eq. (D.1)] indeed comes very close to describing the curve through the plotted points in the figure.² But is it close enough so that in measuring angular distances

² The curve in the figure is not that given by Eq. (D.1) but is a fourth degree polynomial chosen to best fit the plotted points.

FIGURE D.4 (Top) Stars from the same background photo as in Figures D.1, D.2, and D.3. For the two stars α Ori and α Aur the linear distance r and angular distance ϕ to the center of the photo are given. The dashed circles are at $\phi = 5^\circ, 10^\circ, 15^\circ, \dots$ (Bottom) The same stars but with each star plotted at its corresponding point (r, ϕ) . The curve $\phi(r)$ through these points describes the behavior of the lens.

we can use the rectilinear approximation and dispense with the star triangle method? If we use the rectilinear approximation to find the angular separation between two stars, we find that we can get errors up to about half a degree. (Stars near opposite corners of the photo give errors of about a degree, but this is the worst case, and it is not a case that would be expected to arise in measuring halo photos.) Half degree errors are too much for our purposes, especially since we always have to contend with an additional uncertainty of half a degree or more in locating the halo edge. So for our 20mm lens the rectilinear approximation is not good enough. For other lenses the rectilinear approximation may be better or worse, but it is hard to know in advance how good it will be, since lens manufacturers are reluctant to divulge this information. We seem to be stuck with something like the star triangle method.

More Crystallography

This appendix contains derivations of the crystallographic facts that were used in Chapters 7, 9, and 11. In discussing crystallography, our only goal is to get some idea of what faces might be likely on an ice crystal. What we present here is therefore only a fragment of the subject of crystallography as a whole. For much more see the books by Buerger [11] and Senechal [67].

Lattice, basis, dual basis

Associated with a crystal is a 3-dimensional *lattice*, which consists of all integral linear combinations $n_1\mathbf{v}_1 + n_2\mathbf{v}_2 + n_3\mathbf{v}_3$ of three independent vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ in 3-space. The vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are a *lattice basis* for the lattice.

A *lattice plane* is a plane that passes through three non-collinear lattice points. Thinking of the lattice points as sites where atoms can be added to the crystal during its growth, we regard the lattice planes as being the possible crystal faces. Not all of the lattice planes, however, are equally likely. All else being equal,¹ the likelihood of a lattice plane being (parallel to) a crystal face is thought to depend on the density of lattice points on it. Equivalently, the likelihood depends on the spacing between the lattice plane and the adjacent parallel lattice plane; the larger the spacing, the more likely the crystal face.²

¹ All else would *not* be equal if, for instance, the internal structure of the crystal had low symmetry. Ice, however, seems to have full hexagonal symmetry, see page 115.

² The principle that likelihood should increase with lattice spacing is known as Bravais' law. This is not the same as the Bravais' law of classical halo theory [31, pp 504–508]. The latter is now mainly of historical interest and is an easy consequence of our geometrical version of the law of refraction (page 38).

The *dual basis* vectors $\mathbf{w}^1, \mathbf{w}^2, \mathbf{w}^3$ will be the key to handling lattice planes and hence crystal faces. The dual basis vectors are defined by

$$\mathbf{w}^i \cdot \mathbf{v}_j = \delta_j^i = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \quad (\text{E.1})$$

The dual basis for ice is calculated at the end of this appendix.

Planes

A plane, whether a lattice plane or not, is determined by its normal vector $\mathbf{N}$ and by a point $\mathbf{p}_0$ on the plane. Like any vector, the normal vector can be written $\mathbf{N} = h\mathbf{w}^1 + k\mathbf{w}^2 + l\mathbf{w}^3$ for some real numbers h, k , and l . A point $\mathbf{p} = x\mathbf{v}_1 + y\mathbf{v}_2 + z\mathbf{v}_3$ is on the plane if and only if the vector from $\mathbf{p}_0$ to $\mathbf{p}$ is perpendicular to $\mathbf{N}$:

$$\begin{aligned} 0 &= \mathbf{N} \cdot (\mathbf{p} - \mathbf{p}_0) \\ &= \mathbf{N} \cdot \mathbf{p} - \mathbf{N} \cdot \mathbf{p}_0 \\ &= (h\mathbf{w}^1 + k\mathbf{w}^2 + l\mathbf{w}^3) \cdot (x\mathbf{v}_1 + y\mathbf{v}_2 + z\mathbf{v}_3) - \mathbf{N} \cdot \mathbf{p}_0 \\ &= hx + ky + lz - \mathbf{N} \cdot \mathbf{p}_0 \end{aligned} \quad (\text{E.2})$$

That is, the equation of the plane is

$$hx + ky + lz = d, \quad (\text{E.3})$$

where $d = \mathbf{N} \cdot \mathbf{p}_0$. We stress that x, y, z are the coordinates of $\mathbf{p}$ with respect to $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, and that h, k, l are the coordinates of $\mathbf{N}$ with respect to $\mathbf{w}^1, \mathbf{w}^2, \mathbf{w}^3$.

Lattice planes and Miller indices

The equation of a plane not containing the origin can be written $h'x + k'y + l'z = 1$. If the plane is a lattice plane, then it contains three independent lattice points $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$, where $\mathbf{p}_i = x_i\mathbf{v}_1 + y_i\mathbf{v}_2 + z_i\mathbf{v}_3$ for some integers x_i, y_i, z_i . Then

$$h'x_i + k'y_i + l'z_i = 1, \quad i = 1, 2, 3. \quad (\text{E.4})$$

Solving these three linear equations for h', k', l' using, say, Cramer's Rule, shows that h', k', l' are rational and that the plane equation can therefore be rewritten $h''x + k''y + l''z = n''$, where h'', k'', l'', n'' are integers and $n'' > 0$. Since $\mathbf{p}_1$ is on the plane, then $h''x_1 + k''y_1 + l''z_1 = n''$. The integer n'' must therefore be divisible by the greatest common divisor of h'', k'', l'' . Dividing both sides of the plane equation by the greatest common divisor gives $hx + ky + lz = n$, where n is a positive integer and h, k, l are relatively prime integers.

Conversely, if h, k, l are relatively prime integers, then there are integers x_1, y_1, z_1 such that $hx_1 + ky_1 + lz_1 = 1$ [52, page 5]. The lattice point $\mathbf{p}_1 = x_1\mathbf{v}_1 + y_1\mathbf{v}_2 + z_1\mathbf{v}_3$ is therefore on the plane $hx + ky + lz = 1$. So are the two lattice points $\mathbf{p}_2 = \mathbf{p}_1 + k\mathbf{v}_1 - h\mathbf{v}_2$ and $\mathbf{p}_3 = \mathbf{p}_1 + l\mathbf{v}_1 - h\mathbf{v}_3$, and if $h \neq 0$, as we can assume without loss of generality, then $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$ are non-collinear, since $(\mathbf{p}_2 - \mathbf{p}_1) \times (\mathbf{p}_3 - \mathbf{p}_1) \neq \mathbf{0}$. Thus $hx + ky + lz = 1$ is a lattice plane. And so is the plane $hx + ky + lz = n$ since it contains the three non-collinear lattice points $n\mathbf{p}_1, n\mathbf{p}_2, n\mathbf{p}_3$. Thus:

Criterion for lattice plane *A plane not containing the origin is a lattice plane if and only if its equation can be written*

$$hx + ky + lz = n \quad (\text{E.5})$$

for some relatively prime integers h, k, l and some positive integer n .

The integers h, k, l are known as the *Miller indices* of the plane. The vector $\mathbf{N} = h\mathbf{w}^1 + k\mathbf{w}^2 + l\mathbf{w}^3$ is the outward normal to the plane, and the point $\mathbf{p} = x\mathbf{v}_1 + y\mathbf{v}_2 + z\mathbf{v}_3$ with x, y, z satisfying Eq. (E.5), is a typical point on the plane.

To get a more concrete feeling for Miller indices, notice that the lattice plane $hx + ky + lz = 1$, which has Miller indices h, k, l , passes through the three points $(1/h)\mathbf{v}_1, (1/k)\mathbf{v}_2$, and $(1/l)\mathbf{v}_3$. Figure 9.4 illustrates this remark with three examples.

As explained on page 97, Miller indices for ice crystals are usually given as 4-tuples rather than triples.

Spacing of lattice planes

If the Miller indices h, k, l are fixed and if n takes on all positive integral values, then Eq. (E.5), or the equivalent equation $\mathbf{N} \cdot \mathbf{p} = n$, generates all lattice planes having the given Miller indices. The planes are parallel and equally spaced. The spacing between two adjacent planes $\mathbf{N} \cdot \mathbf{p} = n$ and $\mathbf{N} \cdot \mathbf{p} = n + 1$ can be found by taking points $\mathbf{p}_1$ and $\mathbf{p}_2$ on the respective planes and then projecting $\mathbf{p}_2 - \mathbf{p}_1$ onto $\mathbf{N}$. The projection is

$$\frac{\mathbf{N} \cdot (\mathbf{p}_2 - \mathbf{p}_1)}{|\mathbf{N}|} = \frac{\mathbf{N} \cdot \mathbf{p}_2 - \mathbf{N} \cdot \mathbf{p}_1}{|\mathbf{N}|} = \frac{n + 1 - n}{|\mathbf{N}|} = \frac{1}{|h\mathbf{w}^1 + k\mathbf{w}^2 + l\mathbf{w}^3|},$$

that is,

$$\text{lattice plane spacing} = \frac{1}{|h\mathbf{w}^1 + k\mathbf{w}^2 + l\mathbf{w}^3|} \quad (\text{E.6})$$

Earlier we claimed that spacing is correlated with likelihood; the larger the lattice plane spacing, the more likely the crystal face. Table E.1 gives lattice plane spacings for the mineral beryl. Paradoxically, we know more about polyhedral beryl crystals than we do about polyhedral ice crystals. Since we know what the common faces on real beryl crystals are, we can use the table as a partial test of the correlation between spacing and likelihood. From the table it appears that the correlation is good but not perfect.

Table E.2 is the same but for ice instead of beryl. The table should give an indication of what ice crystal faces are likely.³ Only the three most likely faces—in the top three rows of the table—are known to occur.

In earlier chapters we said that the crystal faces with small Miller indices are most likely. That needs qualification. A given lattice will always have many possible lattice bases. Changing from one lattice basis to another will change the Miller indices of a lattice plane, so that a plane with small Miller indices might become a plane with large Miller indices. The plane itself does not change, and neither does the lattice plane spacing. It is the spacing, not the Miller indices, that is the indication of likelihood. For the usual choice of basis, however, large spacing does indeed correspond to small Miller indices, as we said.

Sunagawa [73] has a much more sophisticated discussion of the question of what crystal faces are likely.

Inclination angle x

Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ be our usual lattice basis for ice, and orient the basis so that $\mathbf{v}_3$ is vertical, as in Figure 9.3. For a plane with Miller indices hkl the inclination angle x is then the angle between the plane and a vertical line, which is the same as the angle between its normal $\mathbf{N} = h\mathbf{w}^1 + k\mathbf{w}^2 + l\mathbf{w}^3$ and a horizontal plane. But from Eq. (E.1) we know that $\mathbf{w}^1$ and $\mathbf{w}^2$ are horizontal, and $\mathbf{w}^3$ is vertical, so

$$\tan x = \frac{|l\mathbf{w}^3|}{|h\mathbf{w}^1 + k\mathbf{w}^2|} \quad (\text{E.7})$$

Then from Eq. (E.11),

³ Even without the beryl table, it is clear that spacing is only loosely correlated with likelihood. In Table E.2, for ice, the spacing values for the $\{10\bar{1}0\}$ prism faces and the $\{10\bar{1}1\}$ pyramid faces are nearly the same, yet the former faces are far more common than the latter. And pyramid faces are much more common on small ice crystals than on large ones, yet the spacing should be the same for both, since it is an internal feature.

$$\tan x = \frac{\sqrt{3}}{2} \frac{a}{c} \sqrt{\frac{l^2}{h^2 + hk + k^2}} \quad (\text{E.8})$$

Also see Figures 9.9 and 9.10, which are the special cases $k = 0$ and $h = k$.

Rational Tangents Principle *Suppose $\tan x / \tan x_0 = v/u$ for some small positive integers u and v . Then if x_0 is a crystallographically likely inclination angle, so is x .*

To see why this should be so, observe that if face hkl has inclination x_0 , then from Eq. (E.7) the face $uhukvl$ has inclination x , where $\tan x = (v/u) \tan x_0$. And if h, k, l, u, v are all small integers, then so are uh, uk, vl .⁴

The Rational Tangents Principle is obviously vague, just as “small” is vague.

Calculation of the dual basis

Given a basis $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ we still need to find its dual basis, that is, the basis $\mathbf{w}^1, \mathbf{w}^2, \mathbf{w}^3$ satisfying Eq. (E.1). Letting

$$\mathbf{w}^i = \sum_{k=1}^3 g^{ik} \mathbf{v}_k \quad (\text{E.9})$$

and taking the dot product of both sides of the equation with $\mathbf{v}_j$ gives $\delta_j^i = \sum g^{ik} g_{kj}$, where $g_{kj} = \mathbf{v}_k \cdot \mathbf{v}_j$. Thus the matrix (g^{ij}) is the inverse of the matrix (g_{ij}) . Taking the dot product with $\mathbf{w}^j$ instead of $\mathbf{v}_j$ gives $g^{ij} = \mathbf{w}^i \cdot \mathbf{w}^j$.

For our usual lattice basis for ice (Figure 9.3),

$$(g_{ij}) = \begin{pmatrix} \mathbf{v}_1 \cdot \mathbf{v}_1 & \mathbf{v}_1 \cdot \mathbf{v}_2 & \mathbf{v}_1 \cdot \mathbf{v}_3 \\ \mathbf{v}_2 \cdot \mathbf{v}_1 & \mathbf{v}_2 \cdot \mathbf{v}_2 & \mathbf{v}_2 \cdot \mathbf{v}_3 \\ \mathbf{v}_3 \cdot \mathbf{v}_1 & \mathbf{v}_3 \cdot \mathbf{v}_2 & \mathbf{v}_3 \cdot \mathbf{v}_3 \end{pmatrix} = \begin{pmatrix} a^2 & \frac{-a^2}{2} & 0 \\ \frac{-a^2}{2} & a^2 & 0 \\ 0 & 0 & c^2 \end{pmatrix} \quad (\text{E.10})$$

$$\begin{pmatrix} \mathbf{w}^1 \cdot \mathbf{w}^1 & \mathbf{w}^1 \cdot \mathbf{w}^2 & \mathbf{w}^1 \cdot \mathbf{w}^3 \\ \mathbf{w}^2 \cdot \mathbf{w}^1 & \mathbf{w}^2 \cdot \mathbf{w}^2 & \mathbf{w}^2 \cdot \mathbf{w}^3 \\ \mathbf{w}^3 \cdot \mathbf{w}^1 & \mathbf{w}^3 \cdot \mathbf{w}^2 & \mathbf{w}^3 \cdot \mathbf{w}^3 \end{pmatrix} = (g^{ij}) = (g_{ij})^{-1} = \begin{pmatrix} \frac{4}{3a^2} & \frac{2}{3a^2} & 0 \\ \frac{2}{3a^2} & \frac{4}{3a^2} & 0 \\ 0 & 0 & \frac{1}{c^2} \end{pmatrix} \quad (\text{E.11})$$

⁴ The integers $uhukvl$ might not be relatively prime, but they can be made so by dividing through by their greatest common divisor, and this does not change the argument.

TABLE E.1 Spacing and inclination angle x for lattice planes of the mineral beryl. Lattice planes are given by their Miller indices and are listed in order of decreasing spacing. The five rows marked as common are the five crystallographic forms for beryl given in *Dana's Manual of Mineralogy* [32]. The fact that they appear near the top of the list suggests that there is indeed some correlation between spacing and likelihood; the larger the spacing, the greater the likelihood of getting a beryl crystal face having the given Miller indices. The spacing values are calculated from Eq. (E.6) and then normalized so that the largest is unity. Angle x is calculated from Eq. (E.8). The internal symmetry of beryl is similar to that of ice, but with $c/a = 0.9975$.

Indices	Spacing	Angle x	Form	
0001	1.000	90.0°	basal	(most common)
10 $\bar{1}$ 0	.868	0.0	hex prism I	(most common)
10 $\bar{1}$ 1	.656	41.0	hex dipyramid I	
11 $\bar{2}$ 0	.501	0.0	hex prism II	(less common)
11 $\bar{2}$ 1	.448	26.6	hex dipyramid II	
10 $\bar{1}$ 2	.433	60.1	hex dipyramid I	(less common)
20 $\bar{2}$ 1	.398	23.5	hex dipyramid I	
11 $\bar{2}$ 2	.354	45.1	hex dipyramid II	(less common)
21 $\bar{3}$ 0	.328	0.0	dihex prism	
21 $\bar{3}$ 1	.312	18.2	dihex dipyramid	
10 $\bar{1}$ 3	.311	69.0	hex dipyramid I	
30 $\bar{3}$ 1	.278	16.1	hex dipyramid I	
11 $\bar{2}$ 3	.278	56.4	hex dipyramid II	
...	...	...		

TABLE E.2 Spacing and angle x for lattice planes of ice. This table is similar to Table E.1 but with $c/a = 1.63$, the modern value for the c/a ratio of ice. The spacing is thought to give a rough indication of the likelihood of getting an ice crystal face having the given Miller indices; the larger the spacing, the more likely the face. The table suggests that Steinmetz and Weickmann were using the most likely pyramid faces but that Humphreys was resorting to highly unlikely crystal faces (Chapter 11).

Indices	Spacing	Angle x	Form
0001	1.000	90.0°	basal
10 $\bar{1}$ 0	.531	0.0	hex prism I
10 $\bar{1}$ 1	.469	28.0	hex dipyr amid I (Steinmetz and Weickmann)
10 $\bar{1}$ 2	.364	46.7	hex dipyr amid I
11 $\bar{2}$ 0	.307	0.0	hex prism II
11 $\bar{2}$ 1	.293	17.1	hex dipyr amid II
10 $\bar{1}$ 3	.282	57.9	hex dipyr amid I
11 $\bar{2}$ 2	.261	31.5	hex dipyr amid II
20 $\bar{2}$ 1	.257	14.9	hex dipyr amid I
10 $\bar{1}$ 4	.226	64.8	hex dipyr amid I
11 $\bar{2}$ 3	.226	42.6	hex dipyr amid II
20 $\bar{2}$ 3	.208	38.6	hex dipyr amid I
21 $\bar{3}$ 0	.201	0.0	dihex prism
21 $\bar{3}$ 1	.197	11.4	dihex dipyr amid
11 $\bar{2}$ 4	.194	50.8	hex dipyr amid II
10 $\bar{1}$ 5	.187	69.4	hex dipyr amid I
21 $\bar{3}$ 2	.186	21.9	dihex dipyr amid
30 $\bar{3}$ 1	.174	10.0	hex dipyr amid I
21 $\bar{3}$ 3	.172	31.1	dihex dipyr amid
11 $\bar{2}$ 5	.168	56.9	hex dipyr amid II
30 $\bar{3}$ 2	.167	19.5	hex dipyr amid I
20 $\bar{2}$ 5	.160	53.0	hex dipyr amid I
10 $\bar{1}$ 6	.159	72.6	hex dipyr amid I
21 $\bar{3}$ 4	.157	38.8	dihex dipyr amid
22 $\bar{4}$ 1	.152	8.7	hex dipyr amid II
31 $\bar{4}$ 0	.147	0.0	dihex prism
11 $\bar{2}$ 6	.146	61.5	hex dipyr amid II
31 $\bar{4}$ 1	.146	8.4	dihex dipyr amid
30 $\bar{3}$ 4	.145	35.3	hex dipyr amid I
21 $\bar{3}$ 5	.142	45.1	dihex dipyr amid
31 $\bar{4}$ 2	.141	16.4	dihex dipyr amid
22 $\bar{4}$ 3	.139	24.7	hex dipyr amid II (Humphreys)
...	...	...	
40 $\bar{4}$ 3	.123	21.7	hex dipyr amid I (Humphreys?)

TABLE E.3 Same as Table E.2 except that here $c/a = \sqrt{3/8} = 0.612$, the value for which the Clarke rhombohedral faces are the $\{10\bar{1}1\}$ faces. Names in parentheses indicate whether Bravais, Besson, or Visser used faces with the given Miller indices to explain odd radius halos (Chapter 11).

Indices	Spacing	Angle x	Form	
10 $\bar{1}0$	1.000	0.0°	hex prism I	
0001	.707	90.0	basal	
11 $\bar{2}0$	.577	0.0	hex prism II	
10 $\bar{1}1$	.577	54.7	hex dipyrarnid I	(Bravais, Visser)
11 $\bar{2}1$	.447	39.2	hex dipyrarnid II	
20 $\bar{2}1$	.408	35.3	hex dipyrarnid I	(Visser)
21 $\bar{3}0$	.378	0.0	dihex prism	
21 $\bar{3}1$	.333	28.1	dihex dipyrarnid	
10 $\bar{1}2$	.333	70.5	hex dipyrarnid I	(Bravais, Visser)
30 $\bar{3}1$	.302	25.2	hex dipyrarnid I	(Besson, Visser)
11 $\bar{2}2$	.302	58.5	hex dipyrarnid II	
31 $\bar{4}0$	.277	0.0	dihex prism	
22 $\bar{4}1$	.267	22.2	hex dipyrarnid II	
31 $\bar{4}1$	.258	21.4	dihex dipyrarnid	
21 $\bar{3}2$	.258	46.9	dihex dipyrarnid	
30 $\bar{3}2$	.243	43.3	hex dipyrarnid I	(Bravais, Visser)
40 $\bar{4}1$	.236	19.5	hex dipyrarnid I	(Visser)
32 $\bar{5}0$	.229	0.0	dihex prism	
10 $\bar{1}3$	.229	76.7	hex dipyrarnid I	
41 $\bar{5}0$	.218	0.0	dihex prism	
32 $\bar{5}1$	.218	18.0	dihex dipyrarnid	
31 $\bar{4}2$	.218	38.1	dihex dipyrarnid	
11 $\bar{2}3$	.218	67.8	hex dipyrarnid II	
20 $\bar{2}3$	.213	64.8	hex dipyrarnid I	(Visser)
41 $\bar{5}1$	.209	17.2	dihex dipyrarnid	
21 $\bar{3}3$	.200	58.1	dihex dipyrarnid	
50 $\bar{5}1$	.192	15.8	hex dipyrarnid I	
32 $\bar{5}2$	.192	33.0	dihex dipyrarnid	
41 $\bar{5}2$	.186	31.7	dihex dipyrarnid	
33 $\bar{6}1$	.186	15.2	hex dipyrarnid II	
42 $\bar{6}1$	.183	15.0	dihex dipyrarnid	
22 $\bar{4}3$	.183	50.8	hex dipyrarnid II	
51 $\bar{6}0$	.180	0.0	dihex prism	
31 $\bar{4}3$	.180	49.6	dihex dipyrarnid	
51 $\bar{6}1$	.174	14.3	dihex dipyrarnid	
50 $\bar{5}2$	.174	29.5	hex dipyrarnid I	
10 $\bar{1}4$	.174	80.0	hex dipyrarnid I	(Bravais, Visser)
40 $\bar{4}3$	.171	46.7	hex dipyrarnid I	(Visser)
...	...	...		
30 $\bar{3}4$	.156	62.1	hex dipyrarnid I	(Visser)
...	...	...		
80 $\bar{8}1$	.123	10.0	hex dipyrarnid I	(Visser)
...	...	...		
80 $\bar{8}3$	.110	27.9	hex dipyrarnid I	(Visser)

Living on the (w)Edge

Understanding a halo is sometimes easier from the perspective of the halo-making wedge—this is the insight that underlies the theory of halo poles. In this appendix we use it to show how the pole of a halo determines its contact points, as described in Chapters 14 and 16.

We think of a halo as being generated in a wedge as the wedge takes on various orientations. The following points (vectors) are relevant. Each can be thought of either as a point on the unit sphere, or as an arrow of length one, as explained in Figure 14.1.

- k** The zenith point, that is, the point overhead.
- S** The sun point. It is the light point of the entry ray to the wedge.
- H** The halo point. It is the light point of the exit ray from the wedge.
- D, E** The minimum deviation entry and exit points, respectively. They are the light points of the entry and exit rays for the minimum deviation ray path (Figure F.1).
- P** The spin vector (Chapter 13).

We imagine an observer riding on the wedge as it tumbles through space, and we try to visualize the process of halo formation as seen by the wedge rider. We distinguish between a point as seen normally and the same point as seen by the wedge rider; if the former point is written $\mathbf{V}$, the latter is written $\mathbf{V}_u$. The subscript u refers to the wedge orientation.

What is the relation between $\mathbf{V}$ and $\mathbf{V}_u$ at a particular moment? To the wedge rider, the wedge always looks the same, regardless of the motion of the wedge; to be definite, let's assume that the wedge rider always sees the wedge in standard orientation (Figure 14.2). To see the world as the wedge rider sees it, you therefore

FIGURE F.1 (Left) Minimum deviation ray path and minimum deviation entry and exit points **D** and **E** on the unit sphere. (Right) The corresponding points **D_u** and **E_u**. They represent **D** and **E** as seen from the wedge. To get **D_u** and **E_u** from **D** and **E**, you reorient the entire configuration at left so that the wedge ends up in standard orientation, as shown at the right. Comparison of the two diagrams shows that **D_u** and **E_u** are located on the meridian $y = 0$ at angles of $\Delta_{min}/2$ above and below the equator.

need to reorient yourself so that the wedge appears in standard orientation. Equivalently—and this is what we choose to do—you can instead leave yourself alone and reorient the world so that the wedge is in standard orientation. Thus to get **V_u** from **V**, you imagine the vector **V** to be linked rigidly to the wedge, and you then rotate the entire assembly—vector and wedge—in such a way that the wedge ends up in standard orientation. Wherever **V** ends up—that is **V_u**. Figure 14.3 illustrated **V_u** for **V** = **P**. Thus when we found the pole **P_u** of the halo, we put the wedge in standard orientation; where the spin vector **P** ended up—that was **P_u**. Figure F.1 illustrates **V_u** for **V** = **D** and **V** = **E**.

The points **P_u**, **D_u**, and **E_u** are constant, since **P**, **D**, and **E** are fixed with respect to the wedge, but **P**, **D**, and **E** themselves are apt to depend on the wedge orientation u . The points **k** and **S** are fixed in space and are constant, but **k_u** and **S_u**, which represent the zenith point and sun point as seen by the wedge rider, are apt to depend on u .

As the wedge takes on its various orientations and makes the halo, the point **k_u** traces out the *zenith locus* on the unit sphere. It is the path of the zenith point as seen by the wedge rider.

For a non-contact arc the spin vector **P** points directly up, and so **P** = **k** and then **P_u** = **k_u**. Since **P_u** is independent of the wedge orientation u , so is **k_u**. The zenith locus for a non-contact arc therefore consists of the single point **P_u**, the pole of the arc.

FIGURE F.2 Correspondence between the circular halo (dashed) and the contact circle. The circular halo has center at the sun $\mathbf{S}$ and has angular radius $\Delta_{\min}$. The contact circle has center at the minimum deviation point $\mathbf{D}_u$ and has angular radius equal to the solar zenith angle σ . The point $\mathbf{H}(\tau)$ is the point on the circular halo with bearing τ , and $\mathbf{C}(\tau)$ is the point on the contact circle with bearing τ . According to the criterion for contact, $\mathbf{H}(\tau)$ is a contact point for a given arc at a given solar zenith angle σ if and only if $\mathbf{C}(\tau)$ is on the zenith locus of the arc.

For a contact arc, the spin vector $\mathbf{P}$ is horizontal. Thus $\mathbf{P}$, which varies, remains orthogonal to the fixed point $\mathbf{k}$. But then $\mathbf{k}_u$, which now varies, is orthogonal to the fixed point $\mathbf{P}_u$. The zenith locus of a contact arc is the great circle with center at the pole $\mathbf{P}_u$.

With just a bit more terminology we can express the criterion for contact with the circular halo. The circular halo, or rather its inner boundary, is of course the circle on the unit sphere with center $\mathbf{S}$ and with angular radius $\Delta_{\min}$. The *contact circle* is the circle on the unit sphere with center $\mathbf{D}_u$ and angular radius equal to the solar zenith angle σ . We define $\mathbf{H}(\tau)$ to be the point on the circular halo with bearing τ , and $\mathbf{C}(\tau)$ to be the point on the contact circle with bearing τ (Figure F.2). Here we are using the term bearing in the sense of compass bearing, so that, for example, on a clock face the bearing of 3:00 would be 90° .

Criterion for contact *For a given refraction arc and a given solar zenith angle, the point $\mathbf{H}(\tau)$ on the circular halo is a contact point if and only if the point $\mathbf{C}(\tau)$ on the contact circle is on the zenith locus.*

Thus, to find the contact points of the arc at a given solar zenith angle σ , you just see where the contact circle intersects the zenith locus of the arc. The points of intersection correspond to contact points, with the disposition of the intersection points on the contact circle being the same as the disposition of the contact points on the circular halo.

If the halo is a non-contact arc the situation is simple (Figure F.3). The zenith locus consists of the single point $\mathbf{P}_u$, the pole of the arc. The contact circle passes

through $\mathbf{P}_u$ for only one σ , namely, $\sigma = s$, where s as usual is the angular distance from $\mathbf{D}_u$ to $\mathbf{P}_u$. Thus only when σ equals s does the arc contact the circular halo. In that case the direction of the contact point from the sun is the same as the direction of $\mathbf{P}_u$ from $\mathbf{D}_u$. This is as described in Chapter 14.

If the halo is a contact arc then the zenith locus is the great circle with center at $\mathbf{P}_u$. The dependence of the contact points on σ is illustrated in Figure F.4. The dependence is as described in Chapter 16.

FIGURE F.3 Finding the contact point of a non-contact arc. Contact points always correspond to intersection points of the contact circle with the zenith locus, the correspondence being as in Figure F.2. Here a non-contact arc and its associated circular halo (dashed) are shown at left for two different solar zenith angles σ . The contact circle C and zenith locus K are shown in the middle diagram, as seen looking directly down at $\mathbf{D}_u$, and then again at the right, seen in the conventional perspective (Figure F.1, right). The circle C has center $\mathbf{D}_u$ and angular radius σ . Since the arc here is a non-contact arc, K consists of the single point $\mathbf{P}_u$, the pole of the arc. (Top) The case $\sigma \neq s$, where s is the angular distance from $\mathbf{D}_u$ to $\mathbf{P}_u$. The circle C misses K and so there are no contact points. (Bottom) The case $\sigma = s$. The circle C meets K at $\mathbf{P}_u$, so there is a single contact point $\mathbf{H}$, and the direction of $\mathbf{H}$ from the sun $\mathbf{S}$ is the same as the direction of $\mathbf{P}_u$ from $\mathbf{D}_u$. The halo here is the 18° plate arc with wedge $13\ 25$, for which $s = 59^\circ$. It is among the halos shown in Figure 15.6.

The two halos in Figures F.3 and F.4 have the same pole. Comparison of the two figures will explain the contrasting behavior of contact points for contact arcs and non-contact arcs.

To see why the criterion for contact should be correct, refer to Figure F.5. Suppose first that the point $\mathbf{H}(\tau)$ on the circular halo is a halo point $\mathbf{H}$ for some orientation of the wedge. Since the deviation is minimum, then $\mathbf{S}_u$ and $\mathbf{H}_u$ must coincide with $\mathbf{D}_u$ and $\mathbf{E}_u$, respectively. But the two configurations $\mathbf{kSH}$ and $\mathbf{k}_u\mathbf{S}_u\mathbf{H}_u$ are congruent, since they consist of the same three points (zenith point, sun point, and halo point), just seen from different viewpoints. Comparing the two configurations, we see that the angular distance of $\mathbf{k}_u$ from $\mathbf{D}_u$ must be σ , and the bearing of $\mathbf{k}_u$ from $\mathbf{D}_u$ must be τ . That is, $\mathbf{k}_u = \mathbf{C}(\tau)$, and so $\mathbf{C}(\tau)$ is indeed on the zenith locus.

FIGURE F.4 Finding the contact points of a contact arc. Contact points correspond to intersection points $\mathbf{C}_1$ and $\mathbf{C}_2$ of the contact circle C with the zenith locus K . The circle C has center $\mathbf{D}_u$ and radius σ . For a contact arc, K is the great circle with center at the pole $\mathbf{P}_u$ of the arc. So here K is fixed but C shrinks as the solar zenith angle σ decreases. For large σ (*bottom*), that is, for low sun, there are two intersection points $\mathbf{C}_1$ and $\mathbf{C}_2$ of C with K and therefore two contact points. With decreasing σ the points $\mathbf{C}_1$ and $\mathbf{C}_2$ —and therefore the contact points—move in the direction opposite the pole direction, until at $\sigma = 90 - s$ they merge into one as shown. For smaller σ (*top*) the circles C and K do not intersect, and there are no contact points. In each diagram the view is looking directly down on $\mathbf{D}_u$, the same as in the middle diagrams of Figure F.3. The halo here is the 18° column arc with wedge 13 25, for which $s = 59^\circ$. The halo itself is shown in Figure 16.3. The $\Sigma = 40^\circ$ and $\Sigma = 59^\circ$ diagrams in that figure correspond to the $\sigma = 50^\circ$ and $\sigma = 31^\circ$ diagrams here.

FIGURE F.5 Illustrating the criterion for contact. (*Top left*) A halo point $H = H(\tau)$ on the circular halo. The view is from inside the celestial sphere, as halos are seen in reality, and looking directly at the sun S . (*Top right*) The same points H and S , together with the zenith point k , but seen from outside the celestial sphere. The angular distance from k to S is the solar zenith angle σ . The responsible wedge is at the center of the sphere. (*Left*) Same as top right but showing the view from the wedge. That is, the entire configuration—sphere and wedge—has been rotated so that the wedge is now in standard orientation. Because the deviation is minimum, then $S_u = D_u$ and $H_u = E_u$. Since the bearing and angular distance of k_u from D_u are τ and σ , then $k_u = C(\tau)$. Thus $C(\tau)$ is on the zenith locus.

Conversely, suppose the point $\mathbf{C}(\tau)$ on the contact circle is also on the zenith locus. Then $\mathbf{C}(\tau) = \mathbf{k}_u$ for some wedge orientation u . As the wedge rotates about the vertical axis in space,¹ the point $\mathbf{k}_u$ remains fixed, but $\mathbf{S}_u$, which must be at an angular distance of σ from $\mathbf{k}_u$, traces out the circle of radius σ centered at $\mathbf{k}_u$. Since the fixed point $\mathbf{D}_u$ is also at distance σ from $\mathbf{k}_u$, then at some moment the point $\mathbf{S}_u$ coincides with $\mathbf{D}_u$, in which case $\mathbf{H}_u$ coincides with $\mathbf{E}_u$. As before, the triangles $\mathbf{k}_u\mathbf{S}_u\mathbf{H}_u$ and $\mathbf{kSH}$ are congruent, so that $\mathbf{H} = \mathbf{H}(\tau)$. That is, the point $\mathbf{H}(\tau)$ on the circular halo is a halo point.

A careful reading of the preceding two paragraphs will show that the arc mentioned in the statement of the criterion for contact plays no role in the proof. The criterion is really a criterion for deciding when a point on the circular halo locus is in fact lit.

Well, that was a bit heavy. Let's end with something lighter, an exercise that you can do by living on the wedge: See if you can explain why non-contact arcs whose poles are on the equator must have their contact points on the parhelic circle. Hint: redraw Figure F.5 but with the point $\mathbf{k}_u = \mathbf{P}_u$ on the equator.

¹ Thus we are assuming: If a given wedge orientation is allowable, then so is any wedge orientation that results from the given one by a rotation about the vertical axis. This very mild assumption is satisfied by all of the usual crystal orientation classes and, more generally, by any crystal orientation class that is characterized by a spin vector.

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