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April 2, 2019

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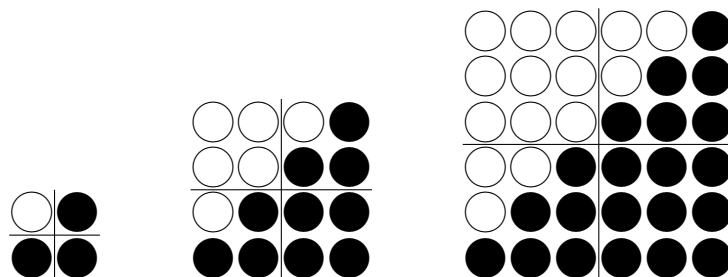
Originally published in: Andrzej Piotrowski (2018). Proof without Words: On Sums of Squares and Triangles, Mathematics Magazine, 91:1, 42, DOI: [10.1080/0025570X.2018.1404885](https://doi.org/10.1080/0025570X.2018.1404885)



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Proof without Words: On Sums of Squares and Triangles

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$$T_k = \sum_{j=1}^k j \quad \implies \quad \sum_{k=1}^{2n} T_k = 4 \sum_{k=1}^n k^2.$$

Remarks: If the diagram above is extended to contain the odd squares too, then it becomes clear that $\sum_{k=1}^n k^2 = \sum_{k=1}^n T_k + \sum_{k=1}^{n-1} T_k$, which can also be derived from the results given in either of the PWW [4] or [5]. Thus,

$$\sum_{k=1}^{2n} T_k = 4 \left(\sum_{k=1}^n T_k + \sum_{k=1}^{n-1} T_k \right) = 4T_n + 8 \sum_{k=1}^{n-1} T_k,$$

which can also be derived by summing the squares of the even integers using the result given in the PWW [2] and the definition of T_n . Finally, the reader is invited to compare this to other PWW involving sums of squares and triangular numbers, especially [1] and [3].

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Summary. We visually display a relationship between sums of squares and the sum of an even number of triangular numbers. Connections to some PWW appearing in the literature are briefly discussed.

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