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Market Allocation of Exhaustive Resources

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The present concern for natural resource conservation has resulted in a surge of interest in the economic theory of the mine. First introduced by Gray (1914), this analysis has generally centered upon the optimal time stream of output for an individual firm in an industry of mines. It was only recently that Richard Gordon attempted to characterize the competitive-market solution for an industry.[1] His result, that a purely competitive-market solution need not be efficient, is odd in that the inefficiency does not arise from any externality or nonconvexity. His technique was to derive the production plan of an individual mine under the assumption that the mine manager maximizes the present value of his resource. This solution was contrasted with that of a socialist manager who maximizes the present value of the resource of the whole industry while acting as a price taker. This note will show that under the usual assumptions, the purely competitive solution is indeed equivalent to that of the price-taking socialist manager. Furthermore, both solutions are economically efficient.

The theory of the mine is an instrument for determining the optimal output pattern over time for a fixed-stock resource, assuming that the resource will be entirely used. Assuming with Gordon that the total stock of the resource is known and of homogeneous quality, and also that production from any mine in any time period does not affect production costs in any later period, the single-mine firm will maximize the present value of the expression

$$\int_{t_1}^{t_2} \pi[q(t), t]e^{-rt}\, dt - \lambda\left[\int_{t_1}^{t_2} q(t)\, dt - K\right],$$

where π denotes the profit function in which $q(t)$, the quantity produced at time t, and time itself serve as arguments. The mining firm's rate of discount is r, and the fixed total stock of the resource it controls is K. Solution of this expression yields the dates for beginning and terminating

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[1] Gordon (1967).

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production as well as the Euler equation, which determines the pattern of production between the beginning and terminating dates. The latter can be simplified to

$$M\pi(t) = MR(t) - MC(t) = \lambda e^{rt},$$

where $M\pi$ is marginal profit, MR is marginal revenue, and MC is marginal production cost. The Lagrangian multiplier λ is usually called a user cost and reflects the fact that whenever the total stock of resources is limited, greater sales today imply fewer sales in the future. This rule tells the firm to allocate its production in such a way that the discounted marginal profit is the same at any instant in time. Thus, marginal profit rises at the rate r and marginal revenue exceeds marginal production cost at every instant of time. The terminal condition requires that at t_2 marginal profit equal average profit; that is, the competitive firm must then operate at the point of minimum average cost.

To this point the analysis is straightforward; confusion arises when Gordon examines the competitive industry. Is the efficient-firm behavior also efficient for the industry? Let us, like Gordon, derive the solution for a socialist manager who maximizes the present value of the total resource while acting as a price taker and see if the decision rules differ from those of an industry of purely competitive mining firms.

The socialist would seek to maximize the present value of the expression

$$\begin{aligned}&\int_0^T W[q_1(t), q_2(t), \ldots, q_n(t), t]e^{-rt}\,dt\\ &-\lambda_1 \int_{{}_1t_1}^{{}_1t_2} q_1(t)\,dt - K_1\\ &\quad\vdots\\ &-\lambda_n \int_{{}_nt_1}^{{}_nt_2} q_n(t)\,dt - K_n,\end{aligned}$$

where the subscripts now indicate the individual mines such that

$$\sum^n q_i(t) = \text{total industry production at } t,$$

$$\sum^n K_i = \text{total industry resource stock.}$$

The integral on the objective function runs over the period of production for the industry as a whole, while the integrals on the constraints each run over the period of production for the particular mine only. The W indicates a welfare function whereby it is assumed that the socialist can maximize utilities and allocate resources efficiently by being a price taker.

The solution to this problem is characterized by a set of $2n$ terminal conditions that give the time intervals for production from each particular

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mine,[2] by the equality of the n resource stock constraints, and by a set of n Euler equations of the following form

$$[P(t) - MC_i(q_i)]e^{-rt} = \lambda_i,$$

where P is the output price, which is unaffected by the output of any individual mine and MC_i is marginal production cost of the ith firm, which is a function only of output of the ith firm. The λ_i is the user cost associated with the ith mine and depends upon the cost function of the mine, its stock of reserves, and on total supply and demand for the industry's output. For example, a mine with low marginal costs and a small stock of reserves would have a large λ_i value relative to a mine with high marginal cost and large reserves. In any period the marginal production costs among mines need not be equalized.

It is in his characterization of the efficient socialist solution that Gordon errs, for he expects the socialist to "minimize production costs in a given period by equating marginal production costs among operating firms."[3] In fact marginal production costs will be equated in only two special cases—when costs are constant, or when all mines are identical in terms of resource stocks and cost curves. In general, the previous equations show that for each mine marginal production cost plus marginal user cost must equal price.

Intuitively, equalization of marginal production costs across mine sites will lower current production costs by increasing output for the more efficient mines and reducing it for those less efficient. However, this situation raises future production costs since the efficient mines are more rapidly depleted, and production must be increased from the higher-cost mines. This cost is what is reflected in the λ_i and when added to the marginal production cost gives the true cost of another unit of output from any mine site. A simple numerical example in the Appendix confirms this.

Equalization of costs across mines occurs in the efficient socialist solution only in the sense of cost defined as the sum of direct production costs and user costs. Gordon does not let the socialist use all information available to him concerning differences among mines with respect to cost functions and resource stocks that is reflected in the differing user costs. Competitive firms would, of course, use this information, and the socialist would be a poor manager if he did not do the same. Gordon incorrectly implies that an industry-wide user cost exists that is the same for each mine.

The solution derived for each mine operated by a socialist manager is identical to the one that would result if the mine were operated by a

[2] Since at least one mine will operate in the first and last production periods, these $2n$ conditions determine the time interval of total production.

[3] Gordon (1967, p. 282).

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purely competitive firm. The socialist requires

$$[P(t) - MC_i(q_i)]e^{-rt} = \lambda_i$$

for each site, and pure competition would require the same result for any single-mine firm.

In each problem, supply and demand for the resource are the same, and each mine site is operating under the same cost function and quantity constraint. The perfectly competitive firm is unable to influence price by its production decision, while the socialist does not attempt to effect a high price, so neither formulation constrains the price of output that arises in the solution. Since that price is derived as a part of the solution to the problem, its pattern must be such that the stock of the resource is exactly exhausted at time T in each case. Thus, since each problem is the identical cost-minimization problem, the output stream determined for each mine, as well as the pattern of price over time, is identical in either case.

Furthermore, as is usual with pure competition, there is no incentive for merger, since the sum of profits from individual mines is the same whether their production schedules are individually or jointly determined. Only if firms become large enough so that price and marginal revenue are no longer equal can merger increase total profits.

The efficiency of the solution characterized above depends upon the nature of the welfare function that the socialist maximizes. If he maximizes his profits subject to the condition that he act as a price taker, the efficient solution will result. In that case, his behavior will be exactly analogous to that of the mining firm that faces a price it cannot affect and produces to the point at which total marginal cost, consisting of marginal production cost and marginal user cost, equals price.

Alternatively, efficiency of the solution can be seen by considering a socialist manager who must rent the n mines yearly from n different owners. The cost of rental of the mines becomes an integral part of the cost of production for the socialist, and he sets the marginal production cost, including mine rental, equal to price to get an efficient solution. The individual mine owner sees his mine as a stock, the present value of which he desires to maximize. The rent he can charge depends upon the costs of production and the amount of resource available at his particular mine site, as well as total supply and demand for the industry's product; at the maximum, it must rise at the rate r. That price at each site should be just equal to the λ_i that the socialist would calculate himself at each site if they were state-owned. Given private ownership, however, the λ_i represent production costs that must be paid, and in this case marginal production costs as viewed by the socialist are equal across all mines within each period.

The analysis easily allows for quality differences within the stock of the resource. Then each competitive-mine firm faces the same problem

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as the socialist manager if one defines the price of output in terms of a given quality of refined ore and defines a cost function for each grade of ore. The same implications follow in that each grade of ore will have a user cost within the firm, and marginal production cost across grades of ore will not be equalized in the profit-maximizing solution. Just as the socialist manager would extract ore simultaneously from mines with different marginal costs when only one grade of ore is present, the competitive-mine firm would extract different grades of ore simultaneously. Thus, in an efficient competitive or socialist industry low-grade ore could be mined at some sites prior to high-grade ore at others.

Thus, the purely competitive solution is efficient, and when Gordon's socialist equates marginal production costs within a period his production scheme will be inefficient.

Appendix

Counterexample to Gordon Rule for Socialists

Assume a very simple world in which the output from the only two mines may be sold in any quantity and in any time period for $5.00. The large mine, B, has a three-unit stock and the small mine, S, has only one unit, but the mines have identical upward-sloping cost curves characterized as follows:

Units of Output	Marginal Cost
1	$2
2	$3
3	$4

Four possible time schemes of production can be compared as to present value at a discount rate of 100 percent. The first three, which equate marginal cost between the mines in each period, are all dominated by the fourth, which implicitly allows user cost to affect the production stream.

	Units Produced by Period			Profit by Period			
	1	2	3	1	2	3	Present Value
Case 1:							
S	1	...	...	$3	...	...	
B	1	2	...	$3	$5	...	$8.50
Case 2:							
S	1	...	...	$3	...	...	
B	1	1	1	$3	$3	$3	$8.25
Case 3:							
S	...	1	...	...	$3	...	
B	2	1	...	$5	$3	...	$8.00
Case 4:							
S	1	...	...	$3	...	...	
B	2	1	...	$5	$3	...	$9.50

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