The complete solution for the problem involves solving first for the homogeneous equation (equal to zero). This complimentary solution is called the "transient" term in the full solution. The transient term solves for the system's response to release from an initial deflection and goes to zero as time goes on. Its behavior depends on the value of ß.

When ß is greater than unity the system is said to be "over-damped". The motion of over-damped systems is described by smooth non-oscillatory curves.

If ß is less than unity, the motion is oscillating and the system is said to be "under-damped". Solutions for under-damped systems are in the form of dampened sinusoids, that is, sinusoidal oscillation with decreasing amplitude as a function of time.

In the case where ß is equal to one, the system is "critically damped". This is motion right on the interface between oscillatory and non-oscillatory motion. When considering a point on a pavement, the transient term for the motion of this point would be the resulting motion following the passing of a single traffic load. The particular solution to the equation would model the response to cyclic traffic loading.

If you are considering the motion of a pavement moving along with the wheel, the solution and equation becomes much more complex. Thompson (1963), performed damping analysis relating moving wheel loads on pavements and determined that the pavement in front of a moving wheel load is oscillatory. Behind a moving wheel load he found over-damped curves. His solution was actually even more complicated, involving the solution of a fourth-order differential equation.

Damping Using Stress - Strain Characteristics

The equivalent linear damping (*βeq*) for a non-linear material ratio is defined as:

$$\beta_{eq} = \frac{1}{4\pi}\frac{\Delta W}{W} \qquad \textbf{(B3)}$$

where: **ΔW** is the area inside of a shear stress-strain hysteresis loop for a test cycle (damping energy).

W is the area of the triangle as shown in Figure 2.2 (strain energy) is used for damping analysis in this study.

Soils exhibit non-linear behavior under dynamic loading. For such materials the study of its load-deformation characteristics is the only tool available to assess the energy dissipation properties.

Using this approach it is assumed that the energy dissipated in damping originates in the material of the structural components of the physical system. The damping characteristics may then be determined by studying the stress-strain nature of the material or the load-deformation curves of the structural system under different loading conditions.

Consider a single layer of a continuous system of thickness, H, and density ρ, subjected to cyclic loading. Assume that the constitutive material may be described by a system similar to a Voight model, with shear modulus G and viscosity η. The stress-strain law of the material is written as:

$$\tau = G\gamma + \eta\dot{\gamma} \qquad \text{(B4)}$$

Where τ is the shear stress and γ is linear shear strain.

The continuous system may be represented by a damped oscillator by setting:

$$
\begin{aligned}
u &= \gamma H \\
K &= G/H \\
C &= \eta/H \\
M &= \rho H
\end{aligned}
\qquad \text{(B5)}
$$

Substituting Eq. B5 into a single degree of freedom version of Eq. B1. The results in giving the right side of Eq. B4 so:

$$F \equiv \tau \qquad \text{(B6)}$$

Therefore, in the Voight model there is identity between structural and mechanical behavior. That is, the shear stress variation is identical to the forcing function.

Damping Energy. By definition the damping energy of the system, $\mathbf{\Delta W_s}$, is the total energy absorbed per cycle of loading by the entire specimen. In other words it is measured by the area of the load deformation loop, which may be expressed mathematically by:

$$\Delta W_s = -\oint F du \quad \textbf{(B7)}$$

Similarly, the damping energy per unit volume of the material, ΔW, is the total energy absorbed per cycle of loading per unit volume of material. Its mathematical expression is:

$$\Delta W = -\oint \tau \, d\gamma \quad \textbf{(B8)}$$

As a consequence of Eq. B6:

$$\Delta W_s = H \Delta W \quad \textbf{(B9)}$$

The energy loss per cycle is defined as the area inside the hysteresis loop of stress versus strain. Damping energy is a material property.

Normally in analysis of linear behavior at steady state oscillation the hysteresis loop is considered as an ellipse and the area is found to be:

$$\Delta W = \pi \eta \Omega \gamma_a^2 \tag{B10}$$

where: Ω is the excitation frequency

γ_a is the shear strain amplitude

Similarly, the damping energy of the system has the expression:

$$\Delta W_s = \pi C \Omega u_a^2 \tag{B11}$$

where: u_a is the amplitude of displacement

By setting Eq. B10 and B11 equal according to equation B9 gives:

$$C u_a^2 = \eta H \gamma_a^2 \tag{B12}$$

Eq. B12 can be modified by using the Eq. B5 definition for **u**, in solving for **C** in Equation B2.1 and noting that $\sqrt{KM}$ is equal to $\frac{K}{\omega}$ per Eq. B2.2. The result is:

$$2\beta \frac{K}{\omega} H = \eta \tag{B13}$$

According to Eq. B5, **G = KH**, inserting this into Eq. B13 gives:

$$\frac{2\beta G}{\omega} = \eta \tag{B14}$$

Inserting Eq. B14 into B10 gives:

$$\Delta W = 2\pi \frac{\Omega}{\omega} \beta G \gamma_a^2 \tag{B15}$$

This is the definition for damping energy of an elliptical stress-strain loop.

Strain Energy. The strain energy for the Voight model as shown in Figure 2.1 is the elastic energy stored in the spring of shear modulus (stiffness). In other words the area under the shear modulus line above the shear strain axis. Mathematically the linear strain energy, **W**, is:

$$W = \frac{1}{2} G \gamma_a^2 \qquad \textbf{(B16)}$$

By dividing B16 into B15, one derives a fundamental relationship for the fraction of critical damping to stress-strain characteristics:

$$\frac{\Omega}{\omega} \beta = \frac{1}{4\pi} \frac{\Delta W}{W} \qquad \textbf{(B17)}$$

At resonance $\Omega = \omega$, then we have the linear damping ratio:

$$\beta = \frac{1}{4\pi} \frac{\Delta W}{W} \qquad \textbf{(B18)}$$

Rate Independent, Non-Linear Damping

Some conclusions important to the construction of an appropriate model for soil behavior are:

1. Soil dynamic response is highly non-linear. The Secant modulus decreases while the damping increases with strain amplitude.
2. The damping is rate-independent in the range of frequencies of interest.

For some materials the damping ratio is not a function of the frequency. This type of damping has been called "hysteretic damping" (Lazon, 1968), but since all damping phenomena are associated with hysteresis loop effects, it may be more appropriately called "rate-independent damping" (Martin, 1976).

For materials of this type, the shape of the stress-strain loop is not influenced by the excitation frequency. Such damping may be linear, viscous-type dissipation force (not effected by loading rate) or non-linear.

Generally, rate independent non-linear materials such as soils, display hysteresis loops having sharp corners at their extreme points.

Equivalent Linearization Techniques

A logical method of solving the dynamic response of actual non-linear systems is to replace them conceptually with equivalent visco-linear systems, which solutions are available. Attention was first drawn to this approach by Jacobsen (1930). He concluded that the most useful criterion was the equivalence of energy dissipated (absorbed) per cycle, **ΔW**. The basis for judging the appropriateness of this criterion was a comparison of the amplitudes of steady state resonant vibrations of an equivalent linear system, and a prototype non-linear system.

Accordingly, Eq. B18 becomes Eq. B3 (Eq. 2.3, in the Thesis):

$$\beta_{eq} = \frac{1}{4\pi}\frac{\Delta W}{W}$$

where: **ΔW** is the area in that non-linear shear stress-strain (hysterisis) loop, determined by numeric integration

W is the area of the triangle under the G_{eq} line defined as the slope of the line passing through the points of maximum and minimum shear strain of a given hysterisis loop. The base of the triangle is the compressive shear strain of the cycle, and the height is the compressive shear stress at maximum strain.

For the type of tests performed in this study, **W** is mathematically defined as:

$$W = \frac{1}{2}\frac{\tau_{max}^{2}}{G_{eq}} \qquad \text{(B19)}$$

The shear modulus and shear stress are determined from Young's modulus and measured deviator stresses using definitions from elastic theory, Timoshenko & Goodier (1987), with an assumed Poisson's Ratio of 0.35 for all materials tested.

APPENDIX C

FALLING WEIGHT DEFLECTOMETERS

AND

BACKCALCULATION

Falling Weight Deflectometer - Background

The falling weight deflectometer (FWD) was originally built in Denmark and is a development of a French device, Ullidtz (1987). The force pulse is obtained by dropping a weight on a specially designed spring system. Using Dynatest Model 800 or 8000 FWD's, this produces an impact load of peak force up to 18,000 or 27,000 pounds, respectively, with a duration of 25-30 microseconds. Drop heights may be varied with the FWD to create lesser impact loads which are appropriate for testing unbound layers. Figure C1 shows a Model 8000 FWD.

The falling weights are dropped on a loading plate of 11.8 inch diameter. The duration of the load over this distance corresponds to an approximately 25 mph wheel velocity. Ullidtz instrumented a test road and compared the response of a moving truck wheel load to an FWD impact load. Excellent correlations were found between the two in terms of deflection, vertical stress and vertical strain.

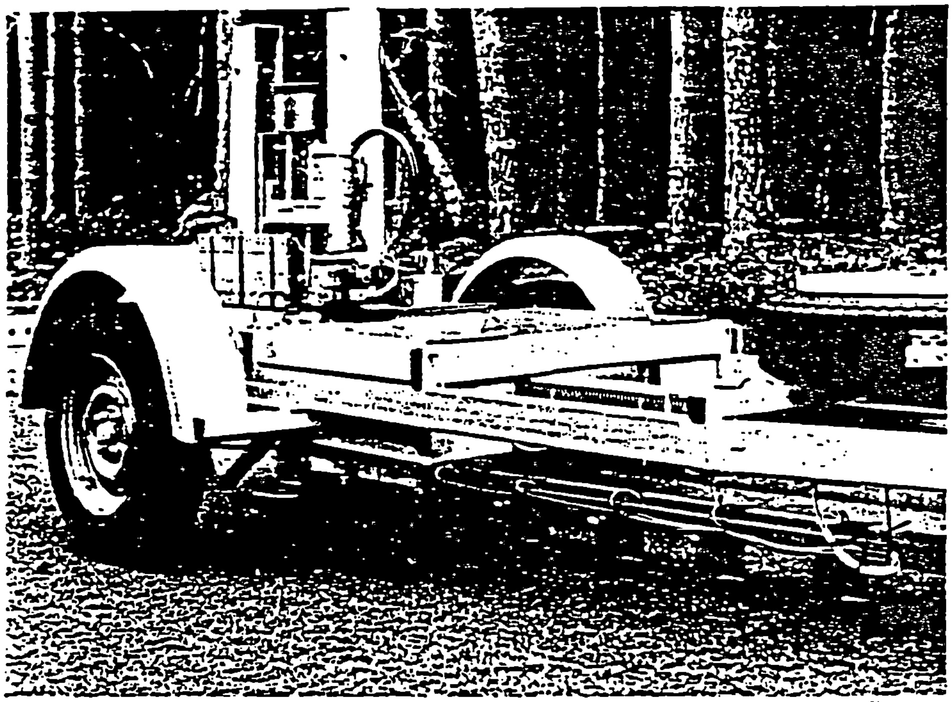

Figure C1 Dynatest Model 8000 Falling Weight Deflectometer

Deflections are measured with geophones at seven different distances from the loading plate. Measurement of deflection are recorded to the nearest 0.01 mils, with typical accuracy of 0.5% $\pm$ 0.04 mils. This accuracy is necessary because the subgrade modulus is determined from the outer sensors which often record deflections of only 0.8 to 1.2 mils, Ullidtz (1987).

The total test sequence is controlled from the drivers seat of the towing vehicle. The results are automatically stored on a computer floppy disk, for later uploading and processing. Figure C2, Gartin (1991), shows graphs of typical FWD data. The data is used for backcalculation of layer moduli which are used in the prediction of stresses and strains at critical locations, using elastic theory, for the purpose of pavement design.

However, as was explained previously, the FWD impact load does not result in shear stress reversal. In this way it does not represent a moving wheel load or tend to generate much pore pressure. Also, the standard FWD sensors can only measure accurately deflections up to 0.080 inch (2 mm). Testing of poorly drained sites often results in readings beyond this range, rendering the data useless.

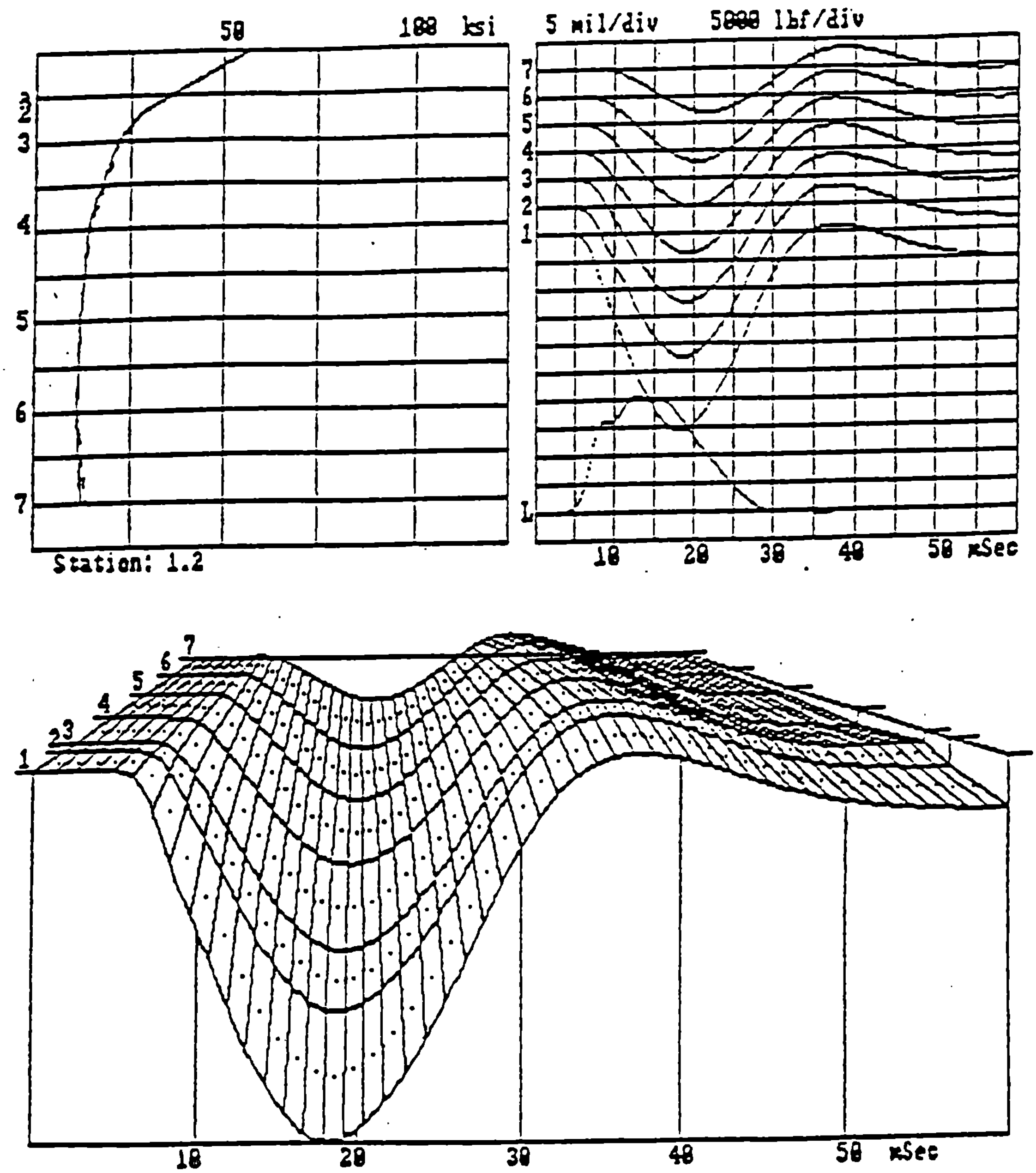

Figure C2 Typical Falling Weight Deflectometer Time History

A) Surface Stress vs. Distance from Load
B) Time vs. Load and Sensor Deflections
C) Time vs. Deflection vs. Distance from Load

NOTE: Sensor 1 is at the Load center, Sensor 7 is offset 47.2"
The others in between.

In poor sections often the post impact response would be non-typical (e.g. deflections not decreasing with distance from the load center). The reason for this type of response is due sometimes to shear failure of the material tested causing upward deflections to be measured adjacent to the loading plate. Data, as this, is of negligible value.

The FWD deflection transducers are unable to measure permanent deformations. They only measure deflections relative to the particular drop. As far as the backcalculation is concerned all deformations are then considered to be recoverable, which could lead to erroneous results should data taken where permanent deformations are occurring be relied upon for design.

Modulus Backcalculation - Background

The computer program, ELMOD, Versions 3.1 and 3.2, were used to backcalculate layer moduli and equivalent depth to stiff layer at field test sites. This program (by Dynatest Engineering A/S) uses the drop stress and measured deflections from the FWD computer disks. ELMOD is an acronym standing for Evaluation of Layer Moduli and Overlay Design.

Moduli are determined using Boussinesq's Equations and Odemark's Method. The principals used in this method are briefly described in the following pages of this section.

In 1885, Boussinesq published a report presenting equations for calculating stresses, strains, and deflections of a homogeneous, isotropic, linear elastic semi-infinite space under a point load. At the centerline under the load the equations for vertical stress (σ_z), vertical strain (ϵ_z) and displacement (d_z) at depth z are:

$$\sigma_z = \frac{3P}{(2\pi z^2)} \quad \text{(C1)}$$

$$\epsilon_z = \frac{(1+\mu(3-2\mu)P}{(2\pi z^2 \mathrm{E})} \quad \text{(C2)}$$

$$d_z = \frac{(1+\mu)(3-2\mu)P}{(2\pi z \mathrm{E})} \quad \text{(C3)}$$

Where: P is the point load

μ is the Poisson's ration

E is Young's Modulus

These equations reveal some interesting facts. Vertical stress is independent of the elastic parameters (μ, E). Stress and strain decrease proportionally to the square of the depth, whereas deflections only decrease linearly with depth. It is also interesting to notice that the strain at a given depth is equal to the deflection divided by the depth.

By integrating the Boussinesq Equations over the loaded area at the surface, stresses and strains can be determined in terms of the surface load stress. In most cases this must be done by numeric integration.

Boussinesq Equations are useful for interpreting results of plate loading tests on subgrade materials. Unfortunately the actual load distribution on soils is neither uniform nor a stiff plate distribution.

According to Ullitz (1987), the problem arising from unknown stress distribution may be solved by measuring the deflections at different distances from the center of the load. It was further reported that comparing the deflections obtained for a distributed load to those obtained for a point load, that for distances greater than twice the radius from the load center, a distributed load may be treated as a point load.

The modulus can then be found from:

$$E = P * \frac{(1-\mu^2)}{[\pi r * d_o(r)]} \qquad \text{(C4)}$$

Where: $d_o(r)$ is the surface deflection at distance r from the center of load.

If the medium tested was a true linear elastic space, the moduli computed at different distances must be identical with FWD testing the modulus values computed measuring the load and deflections at six radial points are computed and graphed. The results give the user an idea of the non-linearity or variation in layers of the system.

Odemark's (1949) Method, is used to transform a system consisting of layers, with different moduli into an equivalent system where all layers have the same modulus, on which Boussinesq equations can be used.

For the stiffness to remain the same in a two layered system:

$$\frac{E_2 I}{(1-\mu_2^2)} = \frac{E_1 I}{(1-\mu_1^2)} \qquad \text{(C5)}$$

Where: I is the moment of inertia, which must remain constant.

This leads to:

$$\frac{h_e^3 E_2}{(1-\mu_2^2)} = \frac{h_1^3 E_1}{(1-\mu_1^2)} \quad (OR) \quad h_e = h_1\left[\left(\frac{E_1}{E_2}\right)\left(\frac{1-\mu_2^2}{1-\mu_1^2}\right)\right]^{1/3} \qquad \text{(C6)}$$

Where: h_e is the equivalent thickness of layer 2 in terms of layer 1

The ELMOD program assumes that all Poisson's Ratio's (μ) are equal to 0.35 so Equation 3.6 reduces to:

$$h_e = h_1\left(\frac{E_1}{E_2}\right)^{1/3} \qquad \text{(C7)}$$

Correction factors may be applied depending on layer thicknesses in relation to the radius of the loaded area and modular ratios. The correction factors are used to improve the agreement with elastic theory.