

# Total and Merchantable Volume of White Spruce in Alaska

Thomas Malone, Jingjing Liang, and Edmond C. Packee Sr.

ABSTRACT

White spruce (*Picea glauca* [Moench] Voss) is a valuable commercial species found in interior and southcentral Alaska. Numerous regional and local volume tables or equations exist; however, no statewide model exists or has been tested for accuracy. There is a demand for an accurate model to determine the cubic-foot volume of white spruce trees in Alaska. Multiple models were developed for white spruce to estimate total and merchantable cubic-foot volume to a 2-, 4-, and 6-in. top. These multiple-entry (diameter and height) models were developed for both inside and outside bark volume from a 6-in. stump. The models were tested on a regional basis at various geographic locations and were shown to be highly accurate. The Alaska models chosen have  $R^2$  at or near 0.99 and mean square error from 0 to 0.16 for all models. These models are shown to be superior to other white spruce models in Alaska.

**Keywords:** volume model, white spruce, Alaska, boreal forest

Individual tree volume models are an essential tool for forest management activities (Avery and Burkhart 2002). They are used to estimate individual tree and total stand volume and to investigate biomass, carbon sequestration, and forest growth and yield (Brackley et al. 2010, Liang 2010, Woodall 2010). Resource managers have been estimating the volume of wood in trees for as long as there has been a forest products industry. The most common units for measuring tree volume are board feet, cubic feet, and cubic meters. Volume estimates were originally presented in volume tables. With the advent of computers, empirical volume equations became more widely used than volume tables. Equations are more convenient and flexible because they can be loaded into spreadsheets and data loggers. The current terminology for volume estimation tools is *volume model*, although the terms *volume table* and *equations* are still in use.

Oettelt (1765) estimated tree volume by comparing it with a cylinder of equal height and basal diameter; Smalian (1837) improved the model by comparing tree sections with the frustums of conoids (Bruce and Max 1989). Today, volume models have been developed for individual tree species or for multiple species in composite models (Avery and Burkhart 2002). Volume models are developed from a variety of explanatory variables such as dbh, height, tree form, and age, with the first two being the most common variables (Schreuder et al. 1993, Huang et al. 2000, Avery and Burkhart 2002).

White spruce (*Picea glauca* [Moench] Voss) is a transcontinental species and the most widely distributed conifer in North America. It has high commercial value (Harlow et al. 1996) and ecological importance (Nienstaedt and Zasada 1990). White spruce is used commercially as structural lumber, house logs, plywood, musical string

instruments, and pulpwood (Nienstaedt and Zasada 1990). It is also a valuable source of firewood and biomass. In Alaska, white spruce accounts for 64–81% of the commercial forest volume of the boreal forest (Hutchison 1967). White spruce is widely distributed in the state from the Canadian border in the east to the Bering Sea in the west and from north of the Brooks Range in the northeast to the Kenai Peninsula and the Bristol Bay area in the southwest (Figure 1). White spruce grows on a variety of soil parent materials including glacial, lacustrine, marine, alluvial (Nienstaedt and Zasada 1990), aeolian, residual, and colluvial (Soil Staff Survey 1999). In Alaska, the largest white spruce are approximately 130 ft tall and can reach 34 in. at dbh. White spruce is a thin bark species (Viereck and Little 2007) that tapers quickly from the stump to the main stem.

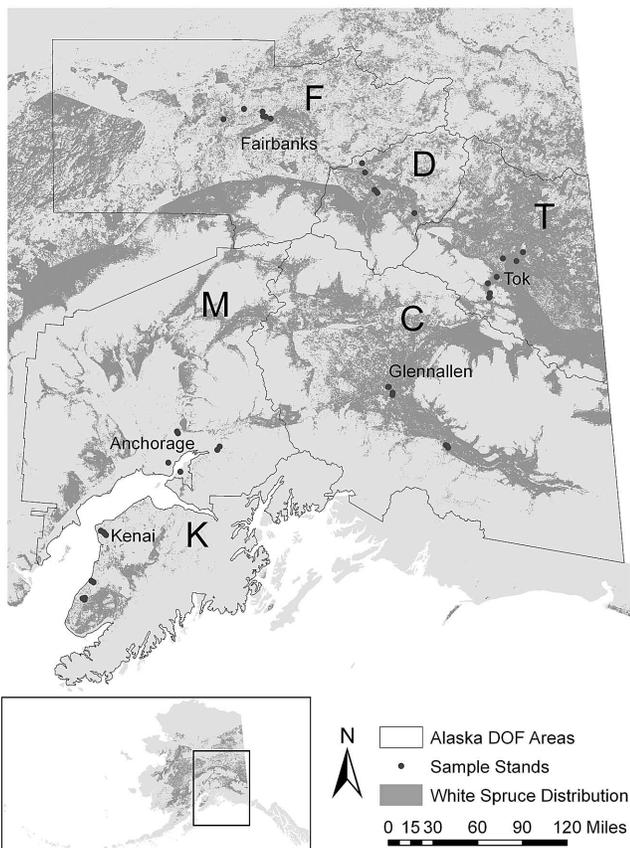
With the rising global and local demand for timber and wood-based energy (State of Alaska 2007), a model to estimate both merchantable and total stem volume of white spruce for forest resources in Alaska is of great need and importance (Liang 2010, 2012, Liang et al. 2011). Existing white spruce volume models in Alaska are of limited applicability because of small sample size or limited sample geographic coverage. The published cubic-foot volume models of Gregory and Haack (1964) were developed with 434 sample trees mostly from interior Alaska, and that of Larson and Winterberger (1988) was developed from 244 white spruce and 43 black spruce trees harvested from the Matanuska and Susitna Valleys. In addition, numerous unpublished volume models based on data sets from small and scattered geographic areas exist.

The objective of this study was to develop a single model to estimate cubic-foot volume of white spruce in Alaska in each of eight categories (total stem volume both outside and inside bark and merchantable volume outside and inside bark to a 2-, 4-, and 6-in. top).

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**Figure 1.** Geographic distribution of the 43 white spruce volume sample sites. Alaska Division of Forestry (DOF) areas: C, Copper River Valley; D, Delta area; F, Fairbanks area; K, Kenai Peninsula; M, Matanuska-Susitna Valleys; T, Tok area (Maisch 2009, Ruefenacht et al. 2008).

## Data and Methods

### Data Collection

A total of 2,016 white spruce trees were sampled from 43 commercial forest stands located throughout six Alaska Department of Forestry (DOF) designated areas (Maisch 2009) of interior and southcentral Alaska: Copper River Valley, Delta, Fairbanks, Kenai Peninsula, Matanuska-Susitna (Mat-Su) Valleys, and Tok (Figure 1). Within each DOF area, sampling locations were randomly selected from stands where trees were being commercially harvested or were available to cut. Sample trees were randomly selected in each stand with four criteria: (1) sample trees must be felled and situated in a safe location for measurement; (2) sample trees were free of major defects from the base to the tip; (3) sample trees must have a dbh no less than 2 in.; and (4) no more than 150 trees would be sampled per stand. Sample trees were representative of dominant, codominant, intermediate, and understory crown classes (Table 1). There was no effort to ensure that there was an even distribution of sample trees by size.

All sample trees were felled and delimited along the top and on one side of the stem to facilitate accurate height and dbh measurements. Data collected for each sample tree were the following: crown class, total height, length of live crown, and diameter outside bark (regardless of abnormalities) every 4 ft along the main stem from a 6-in. high stump to the tip. Length was measured with a tape measure to the nearest 0.1 ft, and diameter was measured with a diameter tape outside bark to the nearest 0.1 in.

**Table 1.** Distribution and number of sample trees by total height and dbh for both the Alaska model data set and the validation (postsample) data set.

dbh (in.)	Total height (ft)										Total
	20	30	40	50	60	70	80	90	100+		
<i>Calibration data set</i>											
2	1	2									3
4	14	47	20	2							83
6	1	30	98	60	12	1					202
8		1	39	135	127	47	5				354
10			5	53	202	148	47	10			465
12				11	87	127	70	30	3		328
14				1	21	69	53	39	19		202
16					4	16	18	24	16		78
18						1	8	17	5		48
20							1	3	10	22	36
22+									1	15	16
Total											1,815
<i>Validation data set</i>											
2											
4		3	5		1						9
6			3	12	10	3					28
8					14	13	6	2			35
10					5	28	13	6			52
12					2	4	13	8	3		30
14						3	5	6	2	3	19
16							1	5	4	3	13
18							1		2	3	6
20							1	2			4
22+										5	5
Total											201

**Table 2.** Sample tree distribution by DOF area.

DOF area	Calibration data set	Validation data set	Total
Copper River Valley	328	36	364
Delta area	323	36	359
Fairbanks area	307	34	341
Kenai Peninsula	353	39	392
Mat-Su Valley	205	23	228
Tok area	299	33	332
Total	1,815	201	2,016

Approximately 10% of the sample trees from each DOF area (Table 2) were randomly selected and reserved to validate model accuracy. A total of 201 validation trees were selected, and the remaining 1,815 trees were used to develop the models.

Stem volume outside bark of each sample tree was calculated as the sum of the volume of each 4-ft section, which was estimated using the Smalian formula (Husch et al. 1993). Stem volume inside bark was estimated by subtracting double-bark thickness from dbh outside bark. Bark thickness was estimated using the white spruce bark thickness model of Malone and Liang (2009), developed from the same geographic areas as these volume data. In estimating the merchantable volume, the top section of each sample tree was set as the first section containing a merchantable top diameter, i.e., 2, 4, or 6 in.

### Model Estimation and Validation

The models to estimate total stem volume outside ( $V_1$ ) and inside ( $V_2$ ) bark were selected from a series of candidate models based on the goodness of fit, level of significance of all coefficients, and residual patterns. The candidate models consisted of various forms of dbh ( $D$ ) and height ( $H$ ) including  $D$ ,  $D^2$ ,  $\ln(D)$ ,  $(DH)$ ,  $(D^2H)$ ,  $(D^2H^2)$ ,  $H$ ,  $H^2$ ,  $(D^{-2}H^{-1})$ , and  $\ln(H)$ . Two different forms of

**Table 3. Alaska models to estimate total and merchantable volume of white spruce in Alaska.**

Model	Estimated parameters	R <sup>2</sup>	CF	MSE
Total stem volume				
Outside bark	$\ln V_1 = -5.7308^{***} + (1.7837 \ln D)^{***} + 1.0613 \ln H^{***}$	0.994	1.005	0.005
Inside bark	$\ln V_2 = -6.1352^{***} + (1.8517 \ln D)^{***} + 1.0691 \ln H^{***}$	0.994	1.005	0.005
Merchantable volume				
Outside bark to a 2-in. top	$M_{1,2} = -0.0734^{***} + 0.9999 V_1^{***} + 0.00003 D + 0.0001 H$	0.999		0.001
Inside bark to a 2-in. top	$M_{2,2} = -0.0720^{***} + 0.9999 V_2^{***} + 0.0001 D + 0.0001 H$	0.999		0.000
Outside bark to a 4-in. top	$M_{1,4} = -0.7100^{***} + 0.9982 V_1^{***} + 0.0354 D^{***} - 0.0028 H^{***}$	0.999		0.016
Inside bark to a 4-in. top	$M_{2,4} = -0.8940^{***} + 0.9963 V_2^{***} + 0.0488 D^{***} - 0.0032 H^{***}$	0.999		0.019
Outside bark to a 6-in. top	$M_{1,6} = -3.1100^{***} + 0.9891 V_1^{***} + 0.2036 D^{***} - 0.0146 H^{***}$	0.999		0.143
Inside bark to a 6-in. top	$M_{2,6} = -3.5510^{***} + 0.9840 V_2^{***} + 0.2541 D^{***} - 0.0198 H^{***}$	0.999		0.158
Merchantable volume (two-stage model)				
Outside bark to a 2-in. top	$M_{1,2} = -0.441 + 0.982 \hat{V}_1^{***} + 0.0128 D + 0.00876 H$	0.989		4.507
Inside bark to a 2-in. top	$M_{2,2} = -0.987^{***} + 0.948 \hat{V}_2^{***} + 0.0825 D^{**} + 0.0138 H^{**}$	0.989		3.602
Outside bark to a 4-in. top	$M_{1,4} = -1.12^{***} + 0.979 \hat{V}_1^{***} + 0.0530 D + 0.0061 H$	0.989		5.193
Inside bark to a 4-in. top	$M_{2,4} = -1.96^{***} + 0.941 \hat{V}_2^{***} + 0.144 D^{***} + 0.0116 H^*$	0.988		3.781
Outside bark to a 6-in. top	$M_{1,6} = -3.86^{***} + 0.964 \hat{V}_1^{***} + 0.2497 D^{***} - 0.00305 H$	0.987		5.978
Inside bark to a 6-in. top	$M_{2,6} = -5.14^{***} + 0.920 \hat{V}_2^{***} + 0.386 D^{***} - 0.00095 H$	0.986		4.498

CF, correction factor for log transformation (only applicable to total stem volume model); MSE, mean squared error;  $V_1$ , total volume outside bark;  $V_2$ , total volume inside bark;  $\ln$ , log transformation;  $D$ , dbh;  $H$ , height;  $M$ , merchantable volume;  $M_1$ , volume outside bark;  $M_2$ , volume inside bark;  $M_2, M_4, M_6$ , merchantable volume to a 2-, 4-, or 6-in. top, respectively.

\*  $P = 0.05-0.01$ .  
 \*\*  $P = 0.011-0.001$ .  
 \*\*\*  $P < 0.001$ .

dependent variables,  $V$  and  $\ln(V)$  were also tested. Although all models tested had high goodness of fit (adjusted  $R^2 > 90\%$ ) and high levels of significance ( $P < 0.05$ ), most models were rejected based on analysis of residuals.

The selected total stem volume model is

$$\begin{aligned} \ln V_1 &= \alpha_0 + \alpha_1 \cdot \ln D + \alpha_2 \cdot \ln H \\ \ln V_2 &= \beta_0 + \beta_1 \cdot \ln D - \beta_2 \cdot \ln H \end{aligned} \quad (1)$$

where  $\alpha$  and  $\beta$  are parameters estimated with the ordinary least squares method (Ryan et al. 2005). To correct the bias introduced by a log transformation (Sprugel 1983), predicted volumes  $V_1$  and  $V_2$  should be multiplied by a correction factor (Baskerville 1972).

Merchantable volume ( $M_{i,j}$ ) was determined by observed total stem volume ( $V_1$  or  $V_2$ ), dbh ( $D$ ), and height ( $H$ ) with a linear model:

$$M_{i,j} = \gamma_0 + \gamma_1 \cdot V_i + \gamma_2 \cdot D + \gamma_3 H \quad (2)$$

where  $M_{i,j}$  represents merchantable volume outside ( $i = 1$ ) and inside ( $i = 2$ ) bark, to a  $j$ -in. top, and  $\gamma$  are parameters to be estimated with ordinary least squares. The observed total volumes,  $V_1$  and  $V_2$ , were used as independent variables for this model.

Because in practice it is often difficult to obtain observed total volume data, we further developed a two-stage merchantable volume model to predicted merchantable volume based on the predicted total stem volume (Wooldrige 2009):

$$M_{i,j} = \delta_0 + \delta_1 V_i + \delta_2 D + \delta_3 H \quad (3)$$

where  $\hat{V}_1$  and  $\hat{V}_2$  are total stem volumes predicted by model 1 and  $\delta$  are coefficients.

Total and merchantable volume models were examined with the coefficient of determination ( $R^2$ ) and residual patterns and were further tested for accuracy and geographic variation. The accuracy of the models chosen was determined by the postsample prediction errors, the difference between the predicted and actual observed total and merchantable volume of the 201 validation sample trees. Predictions of the merchantable volume model to a 4-in. top were compared against observed values and predictions of three other

white spruce volume models used in Alaska (Haack 1963, Beagle 1979, Larson and Winterberger 1988). The 201 validation samples were used to test the hypothesis that a statewide volume model could be accurate for applications across the region and within the six DOF areas. The total stem volume outside-bark model is presented as an example of the accuracy of the Alaska statewide models.

## Results

The model of total stem volume outside bark developed for white spruce in Alaska was

$$\ln V_1 = -5.7308 + (1.7837 \ln D) + (1.0613 \ln H) \quad (4)$$

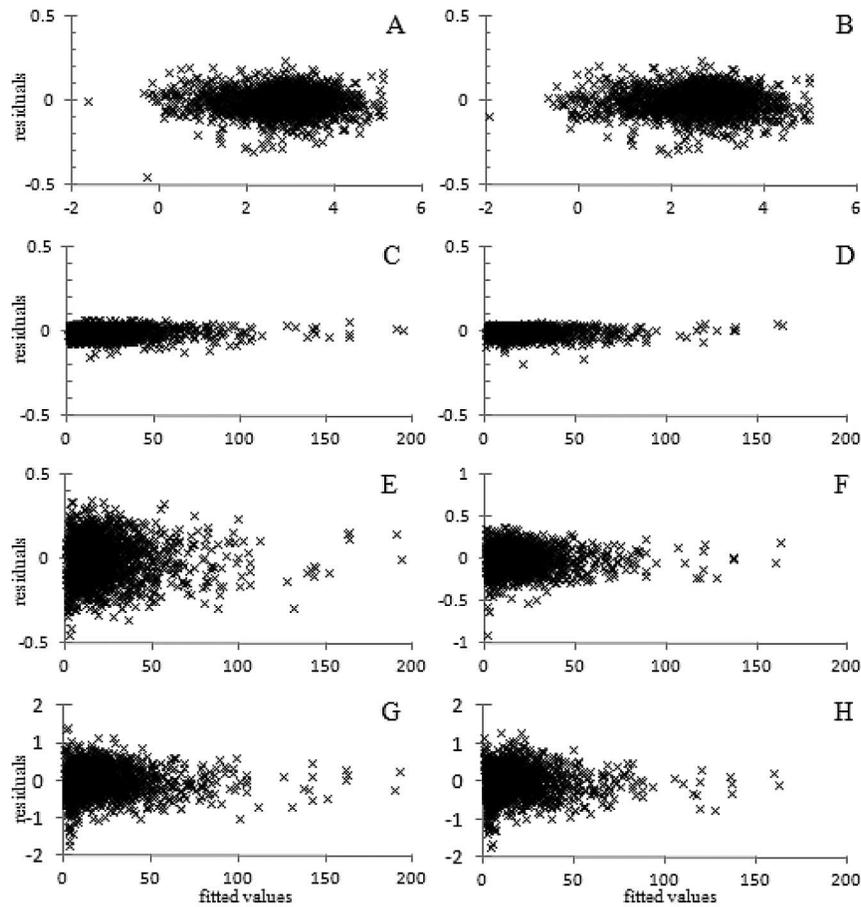
For users who are not familiar with logarithmic transformations, the above model is also presented in exponential form when the correction factor for log transformation has been taken into account:

$$V_1 = 0.00326 D^{1.7837} H^{1.0613} \quad (5)$$

The model of total stem volume inside bark was also developed with the same form (Table 3). The model to estimate merchantable volume outside bark to a 2-in. top was

$$M_{1,2} = -0.0734 + 0.9999 V_1 - 0.00003 D + 0.000094 H \quad (6)$$

The models of merchantable volume inside and outside bark to a 4- and 6-in. top were also developed in the same form (Table 3). All selected total and merchantable models (Table 3) show high coefficients of determination ( $R^2 > 99\%$ ) and high levels of significance ( $P < 0.0005$ ). Residuals of the total stem volume inside and outside bark were normally distributed without any discernible pattern of bias (Figure 2A and B). Residual patterns of the merchantable volume models appear to be more complex (Figure 2C–H), but as the merchantable top diameter increases from 2 to 6 in., the residuals became centered around 0 with a less discernible pattern. There were a few suspicious outlying residuals (Figure 2A and B), but the Cook's distance test ( $P \geq 0.05$ ) indicated that none of them was an influential outlier (Neter et al. 1996, Ryan et al. 2005). This result is expected with such a large sample size.



**Figure 2.** Residual versus fitted values of the total and merchantable volume models of white spruce in Alaska. A. Total stem volume outside bark. B. Total stem volume inside bark. C. Merchantable volume to a 2-in. top outside bark. D. Merchantable volume to a 2-in. top inside bark. E. Merchantable volume to a 4-in. top outside bark. F. Merchantable volume to a 4-in. top inside bark. G. Merchantable volume to a 6-in. top outside bark. H. Merchantable volume to a 6-in. top inside bark.

The accuracy of the merchantable volume model to a 4-in. top inside bark was analyzed by comparing the predicted stem volumes against the observed values of the 201 validation samples. Figure 3 was developed to test the accuracy of the four white spruce volume models with an independent set of postsample data. Three of the models, Haack (1963), Beagle (1979), and Larson and Winterberger (1988), were published previously and the fourth is the Alaska statewide model (this article). The Alaska statewide merchantable volume model was obviously more accurate than the three other volume models that overestimated the merchantable volume of trees of all sizes. For all validation samples, the average predicted Alaska merchantable volumes were all within the 95% confidence interval of the observed mean (Figure 3). The average difference between the mean predicted and the observed stem volumes is  $-0.06 \text{ ft}^3$ , which accounts for  $<0.1\%$  of the observed mean.

Compared with the six DOF individual area models, the Alaska statewide total volume model exhibited no practical or statistical difference in the predictions of the 201 validation samples. Total estimated volumes by dbh using the statewide model and the six individual area models were all similar and fell within the 95% confidence interval of the observed means (Figure 4).

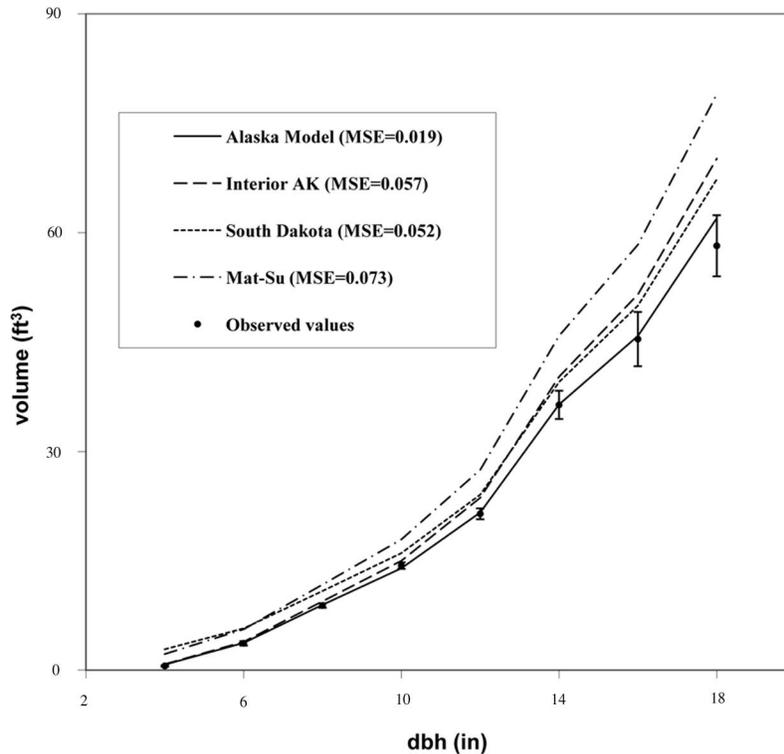
In addition, the Alaska model was tested with the validation samples by DOF area. All the residuals fell within the 95% confidence interval of the observed total volume outside bark, and most of the residuals were less than 1 SD away from the observed values

(Figure 5). Therefore, we found no significant geographic difference in total volume estimation within the State of Alaska, and all the statewide volume models were tested and found to be accurate for all the six DOF areas.

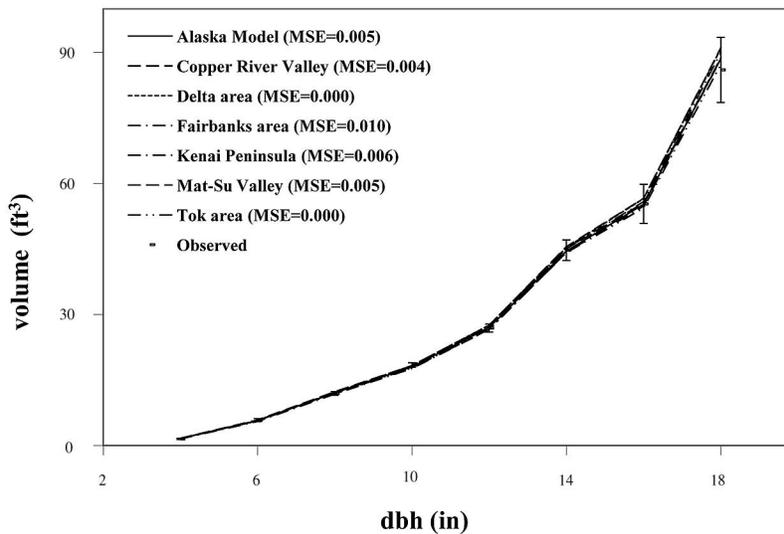
## Discussion and Conclusion

Each of the eight Alaska statewide volume models was tested to be accurate for estimating the cubic-foot volume of white spruce in Alaska. The total stem and merchantable volume models developed in this study display excellent goodness of fit ( $R^2 \geq 0.994$ ) and high levels of significance (Table 3). When tested on the validation sample set, both the total stem and merchantable volume models were accurate, and the prediction errors were not significantly different from 0 (Figure 3). The high coefficient of determination ( $R^2$ ) is a function of the growth pattern of white spruce trees and the sampling method.

White spruce is a medium-size tree that tapers quickly from a small root collar (stump) so that sections of the entire stem are close to the geometric shape of a paraboloid or cone (Husch et al. 1993). In addition, the sample trees were measured in 4-ft lengths so that each section was close to a geometric shape, which lessened the natural variation. This produced a total stem model close to a function of  $D^2H$ , which describes the geometric shape of a cone, thus producing a high  $R^2$ .



**Figure 3.** Mean predicted and observed (with 95% confidence interval) merchantable volume to a 4-in. top inside bark of the 201 validation sample trees. Predictions were obtained with the Alaska model and three other white spruce volume models. Mean square error (MSE) of predicted values is presented for each model. Reference models include the Interior Alaska model (Haack 1963), the South Dakota model (Beagle 1979), and the Alaska Mat-Su model (Larson and Winterberger 1988).

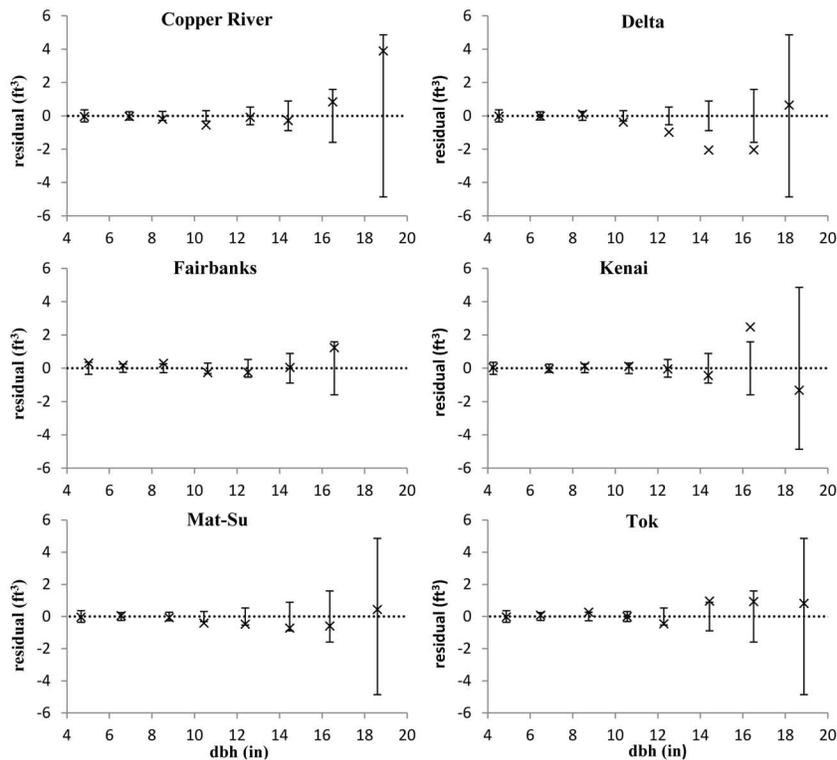


**Figure 4.** Mean predicted and observed total volume outside bark ( $\text{ft}^3$ ) of the 201 validation sample trees, with 95% confidence interval (vertical bars) of the observations. Mean square error (MSE) of predicted values is presented for each model. Reference models are in the same form as the Alaska model but are calibrated only from sample data of each DOF area.

Compared with three existing volume models for white spruce, the Alaska merchantable model was of superior accuracy. The South Dakota (Beagle 1979) and the Mat-Su (Larson and Winterberger 1988) models overestimate merchantable volume of all tree sizes. The interior Alaska model developed by Haack (1963) was accurate for small diameter trees, but the error increased with increasing dbh (Figure 3). All models included in this study overestimate the volume of large diameter trees. There are few white spruce trees in

northern Alaska that reach a dbh of 18 in. (Table 1; Malone and Liang 2009). In this study, the sample size of large trees is small, which accounts for a decrease in accurate volume estimates of large white spruce trees, although the Alaska model estimates are within the confidence interval.

Most existing tree volume models estimate volume from a 1-ft high stump. Modern logging equipment, such as feller bunchers, cut stumps at 6 in. above the ground, which captures more volume at



**Figure 5. Test of model accuracy on validation samples by DOF areas. Residuals are the difference between predicted and observed total volume outside bark (ft<sup>3</sup>). Vertical bars represent 1 SD of the observed values.**

harvest. This difference in harvestable volume can be significant because about half of the volume of a white spruce tree is in the first 16-ft log; in a random subsample of 50 trees used in this study, 54% of the volume was found in the first log. To reflect that increase in harvestable volume due to modern logging equipment, we estimated total and merchantable volume from a 6-in. stump.

It should be noted that to estimate merchantable volume inside or outside bark, total stem volume must be obtained first. If actual total stem volume data is available, merchantable volume model 2 is recommended because of its high goodness of fit. Otherwise, total stem volume predicted by model 1 should be used to replace the observed volume and in this case the two-stage merchantable volume model 3 should replace model 2 in estimating merchantable volume to avoid bias estimation.

A Kuskokwim River Valley white spruce model (Dippold and Farr 1971) estimates merchantable volume to a 4-in. top outside bark. Compared with the merchantable volume model developed in this study, the Kuskokwim River Valley model underestimates the merchantable volume of the validation sample. The difference between the two models may be largely due to different sampling procedures instead of geographic difference. The Kuskokwim data set contains only trees greater than 5-in. dbh, and volume was calculated from a 1-ft stump instead of a 6-in. stump used for our statewide model. The trees from the Kuskokwim data set were measured in 8.15-ft sections, and the Smalian formula was used to calculate volume. The Smalian formula estimates volume accurately for short logs such as the 4-ft sections used in our study. However, the Smalian formula is less accurate for estimating volume of logs longer than 4 ft (Husch et al. 1993, Avery and Burkhart 2002).

This study provides cubic-foot volume estimates of standing white spruce trees for Alaska; many existing models present board-

foot volume. Cubic-foot volume or a volume-weight measurement is more commonly used in the contemporary forest products industry, which is capable of utilizing the entire tree instead of only solid sawn lumber (Nilsson and Wernius 1976). A board-foot scale does not account for bark, sawdust, slabs, and small diameter wood. In addition, for trees sold on an international market, cubic-foot volume can be easily and accurately converted to metric scale by multiplying ( $1 \text{ ft}^3 \times 0.02832 = 1 \text{ m}^3$ ). Board-foot volume can be converted to cubic foot or metric scale but only on an individual stem basis, which is time-consuming.

The Alaska models were developed with data collected from 2,016 trees of various sizes and from 43 forest stands throughout interior and southcentral Alaska. These models provide a simple and accurate tool for researchers and resource managers to estimate white spruce total and merchantable cubic-foot volume, both inside or outside bark from a 6-in. stump.

Additional research should be directed to development of cubic foot/weight relationships because much of the North American and international forest products industry uses weight and conversion factors for solid wood cubic content in relationship to weight of wood and bark.

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