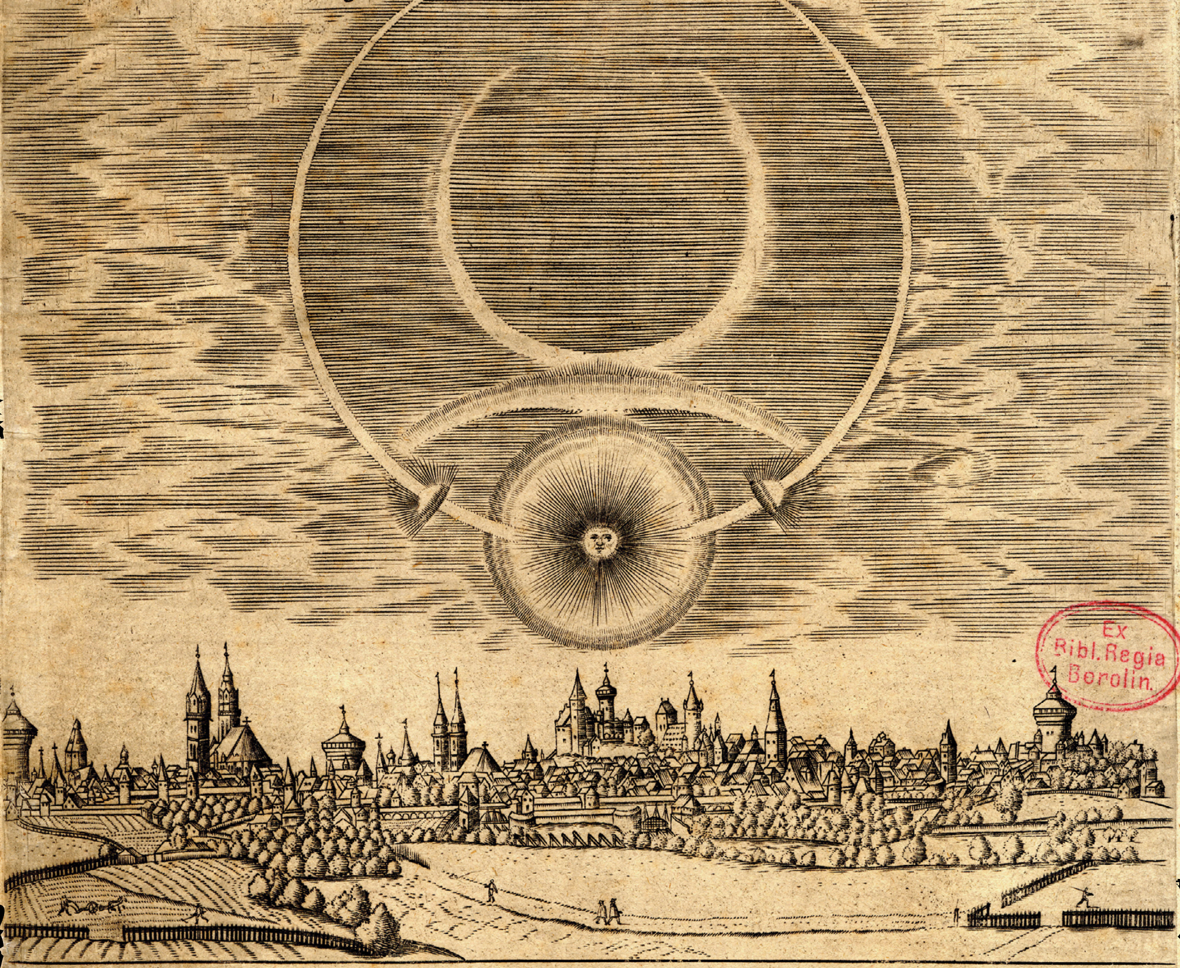
Παρηλια, oder vngewöhnliche Zeichen vmb die Sonnen, wie sie vnter dem Nürnbergischen Horizonte den 19 Aprilis Anno 1630 etliche stünd vor mittag, von menniglich gesehen worden



Ò Gott, ô fromer Gott was zeigstu vns in gnaden!
Wie trewlich warnest vns vor allgemeinen schaden!
Will es dann helfen nicht, so ist das schwerd gewetzt,
Vnd das tödlich geschoß itzt vff die senn gesetzt.

Zwar die natür hatt aüch hierinn ihre vrsachen;
Aber deß halb will ich kein spottwerck daraüß machen.
Ein vngewöhnlich ding ist solch himmels figür;
Gott droht dürch wort v. werck: Gott droht dürch die natur.

M.I.S.

Ioh: pfann Scup. et E.

CHAPTER 3

The Beginnings of Halo Science

Presumably there were halos long before people. We do know that humans have long been aware of halos. In *The Rainbow: From Myth to Mathematics* [8], Carl Boyer writes of "cuneiform tablets of the Sumerian-Babylonian culture of four or five thousand years ago, in which names are given to the smaller halo radius of 22° (tarbasu) and the larger one of 45° radius (supuru)." Boyer mentions halos in connection with Aristotle (384–322 BC), Alexander (c. 200 AD?), Alhazen (c. 965–1039), and Theodoric of Freiberg (c. 1300), among others.

Some ancient paintings and even petroglyphs have been interpreted as depicting halo displays [27, 63]. The oldest unequivocal halo depictions that we know of, however, are not so old, with ages measured in centuries rather than millennia. The exquisite illustration in Figure 3.1 is from the early seventeenth century. It is by no means one of the oldest halo illustrations, but it is one that is easy to interpret, leaving no doubt that what is depicted is a halo display. The circle centered on the sun in the figure is of course the 22° halo, and outside it to the left and right are the parhelia. Just above the 22° halo is the tangent arc, and above the tangent arc is the rare Wegener arc. The large circle passing through the sun and through both parhelia is the parhelic circle. The artist had to abuse the perspective in order to show the entire sky and still include the city in the foreground, but otherwise the representation is excellent.

Although some of the common halos have been known since antiquity, satisfactory explanations of them were a long time in coming. René Descartes, for

FIGURE 3.1 Halo display in Nürnberg, April 19, 1630. German speakers will learn from the verse at the bottom that halos are not to be taken lightly. Staatsbibliothek zu Berlin – Preußischer Kulturbesitz YA 6192 kl, reproduced with permission.

instance, in his *Meteorology* [14] of 1637, attributed the parhelia to the presence of a giant ring of ice in the sky. Although his explanation sounds ludicrous to us today, Descartes himself apparently saw no problem with it; he concluded his *Meteorology* by saying that "I hope that those who have understood all that has been said in this treatise will, in future, see nothing in the clouds whose cause they cannot easily understand, nor anything which gives them any reason to marvel." Yet Descartes was not stupid, and his explanation of the parhelia probably says more about the nearly complete ignorance of the atmosphere at that time, and about the absence of any competing explanation for the parhelia, than it does about his intelligence.

In any case, Descartes' explanation of the 22° halo turned out to be more fruitful. He supposed that it formed in suitably shaped snow crystals high in the atmosphere; the crystals were thick in the middle and then tapered toward the edges. Such crystals could indeed make a 22° halo if they were shaped just right, but Descartes gave no details, and it is not clear to us exactly what he had in mind. The theory that he presented was purely qualitative, and we do not know whether he actually made the calculations that would have made his explanation more convincing. Nevertheless, to suggest that halos were due to ice particles was an important step. It prompted at least one other person to think along similar lines.

That person was Christiaan Huygens, and it seems to be Huygens who came up with the first quantitative explanation for halos. In his *Traité des Couronnes et des Parhélies* [33], written about 1662, he showed how the parhelia could arise in transparent water or ice cylinders having opaque cylindrical cores (Figure 3.2). Huygens assumed that the cylinders were floating in the air with their axes vertical and that the ratio m of the core diameter to the outer diameter was 0.48. Under these assumptions, you do indeed get something resembling parhelia, as shown in Figures 3.3 and 3.4.

Huygens also showed how these same cylinders, if floating with their axes horizontal instead of vertical, would give illumination from the regions above and below the 22° halo rather than from the regions to the right and left of it; such cylinders would thus explain the tangent arc. To account for the 22° halo, Huygens replaced the cylinders with spheres, each sphere being transparent and having an opaque spherical core, again with $m = 0.48$. For the circumzenith arc he used horizontal cylinders as for the tangent arc, but with m changed from 0.48 to 0.68. And for the 46° halo he used spheres, also with $m = 0.68$.[1]

[1] At the time that Huygens began his halo studies, the angular radii of the 22° and 46° halos were not known accurately. Huygens and others took them to be 22.5° and 45°, so that the angular diameters had the appealing values of 45° and 90°. Huygens originally used $m = 0.48$ and $m = 0.68$, from which he obtained the desired angular radii. When better measurements became available for the 22° halo, he replaced $m = 0.48$ with $m = 0.473$.

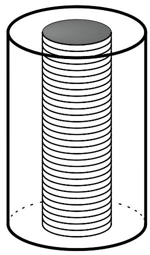


FIGURE 3.2 Huygens' parhelion maker, a transparent cylinder with an opaque cylindrical core. The sky was supposed to be full of such cylinders, all falling with their axes vertical, as here.

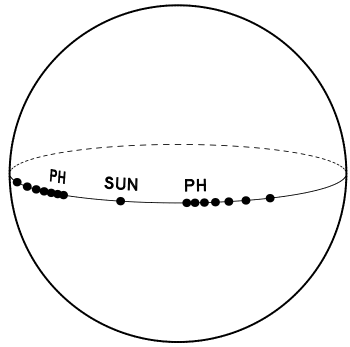


FIGURE 3.3 Celestial sphere with right and left parhelia (**PH**) arising in Huygens' cylinders. Here the sun and the parhelia are on the horizon. The tail of each parhelion is longer and brighter than in reality, but the crucial inner edge of each parhelion is in the correct position. The figure was made from a diagram like the middle one in Figure 3.4 but with more sun rays. Each dot corresponds to an outgoing ray from the cylinder.

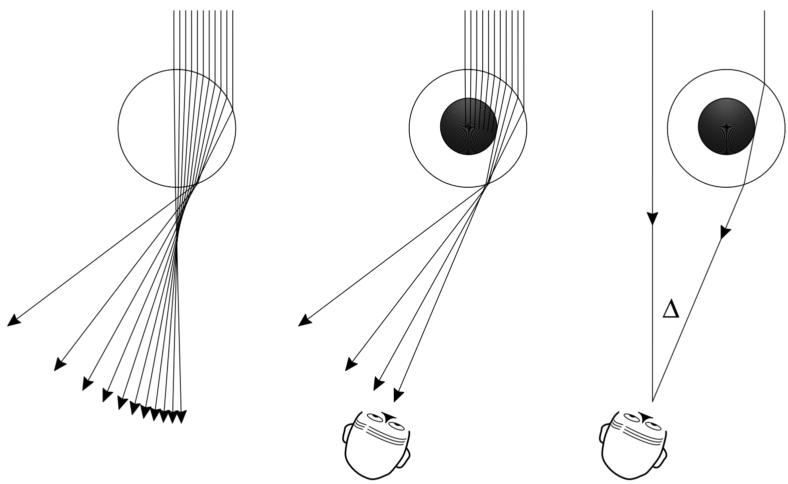


FIGURE 3.4 Light rays forming the right parhelion according to Huygens. This is a top view looking straight down on three vertical cylinders, which therefore appear here as disks. The sun is on the horizon, and all light rays are in the plane of the paper. (*Left*) Ray paths through a transparent cylinder. The deviation Δ between the incoming and outgoing rays varies from 0° upward. (*Middle*) Same but with the addition of an opaque cylindrical core. Now Δ varies from 22° upward, with the densest concentration of rays at 22°. (*Right*) Same but showing only the least deviated ray together with a sun ray reaching the observer. The observer looks in a direction Δ = 22° to the right of the sun to see the inner edge of the parhelion.

Edme Mariotte was the first to attribute halos to prismatic ice crystals. In his *De la nature des couleurs* [45] in 1681, he showed how the 22° halo could arise in randomly oriented columnar crystals, each crystal being in the shape of an equilateral triangular prism. Mariotte calculated light ray paths through crystals having various orientations and found a value of 22°50′ for the minimum deviation between the incoming ray from the sun and the outgoing ray. A sky full of such crystals would show an abrupt increase in brightness—darker toward the sun, lighter away from it—at an angular distance of 22°50′ from the sun; this was the 22° halo. To explain the parhelia, Mariotte used the same crystals but oriented them with their axes vertical.[2]

Mariotte's theory convinced hardly anyone. Did it even get a serious look from his contemporaries? We do not know, but if it did, it seems to have been soon largely forgotten. Smith's *Opticks* [69] of 1738, for instance, contains a twenty-nine page English translation of Huygens' work but never mentions Mariotte. Thomas Young, writing in 1807, claimed that by that time Mariotte's ideas on halos had been "almost entirely abandoned and forgotten."

Today Mariotte is not exactly a household name, but he was well known and highly regarded in his day. His theory of color, for example, is said [8] to have received more attention than that of his contemporary Isaac Newton. Today Mariotte is probably best remembered for his independent discovery of Boyle's Law, also known as Mariotte's Law. Mariotte's ideas on halos had been presented to the French Academy of Sciences in 1679 and had been published in 1681, then republished in Mariotte's collected works in 1717 and 1740. So they were available, whether or not anyone was paying attention.

Mariotte's ideas on halos were eventually revived—at the beginning of the nineteenth century, by Thomas Young [91] in England and Giambatista Venturi [83] in Italy. Since that time, Mariotte's basic idea—that halos form in polyhedral ice crystals in the atmosphere—has been universally accepted.

An imaginary debate

Today Huygens' theory, with its ad hoc cylindrical and spherical particles, seems preposterous. Yet it was apparently the prevailing theory of halos for more than a century. How could it have happened? We are not historians of science and we are not competent to say how it happened in reality, but we can at least muster the arguments for the two sides and try to imagine how a Huygens vs. Mariotte debate might logically have played out, had it ever come to pass. In doing so, we will be forced to think about how we know what we know.

[2] Mariotte's calculation of ray paths for the 22° halo applies more properly to the parhelia (with the sun on the horizon) than to the 22° halo, since the paths that he considered all lay in a plane perpendicular to the crystal axis.

Let us therefore consider the Huygens vs. Mariotte issue on its merits, but from the perspective of the eighteenth century. Just how preposterous were Huygens' ideas, and how preposterous were those of Mariotte?

For evidence for his layered spheres and cylinders, Huygens quoted Descartes' *Meteorology:* "…the outside of each grain of this sleet is usually composed of continuous and transparent ice, yet it has a bit of snow inside." Mariotte, on the other hand, pointed to the fern-like branches that occur on some snow crystals. He had apparently convinced himself that the tiniest extremities on these branches were triangular columns. Similar columns, perhaps unattached to any larger crystal, were supposed to be floating high in the atmosphere and causing the halos.

Today we know that Huygens' precisely layered spheres and cylinders do not exist, and therefore no one could have seen one. But we also know that triangular columns are so rare that nobody at the time would have seen one of them either. We do not find in the old literature any mention even of hexagonal columns, which are much more common than triangular columns and which are nearly equivalent in their optical effects. Robert Hooke had observed snow and frost crystals through the microscope and had published exquisite drawings of them in his popular *Micrographia* [28] in 1665, but there was nothing in his drawings to suggest the existence of prismatic columnar crystals, either triangular or hexagonal. Johannes Kepler [35] in 1611, Descartes [14] in 1637, Frederick Martens [46] in 1694, John Nettis [51] in 1756, and Johann Carl Wilke [90] in 1761 also described or drew snow crystals, but again there was no mention of hexagonal columns, though Descartes and Wilke had come close (Figure 3.5).

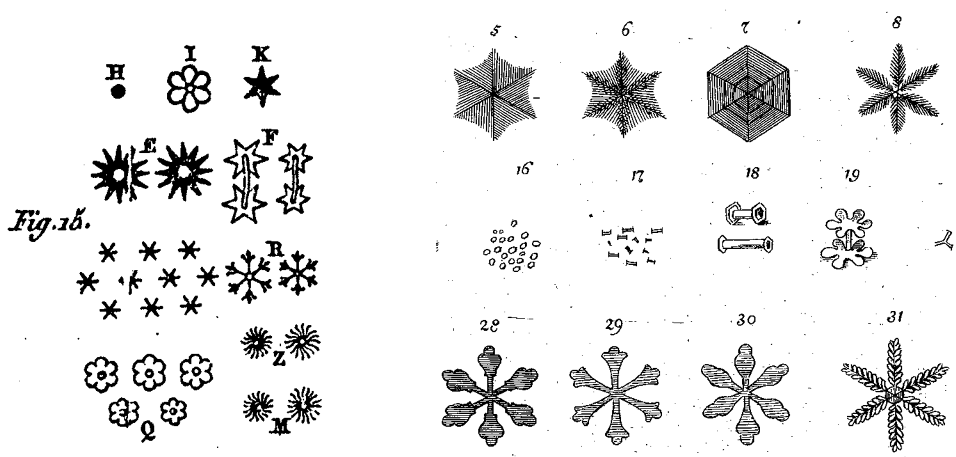


FIGURE 3.5 Ice crystal drawings by Descartes in 1637 at left, and Wilke in 1761 at right. Descartes' diagram **F** and Wilke's diagrams 17, 18, and 19 depict what are almost certainly capped column crystals—hexagonal columns with a hexagonal plate on each end. At the time there seems to have been no awareness of hexagonal columns, and the columnar parts of the capped columns here were understandably misinterpreted as cylinders.

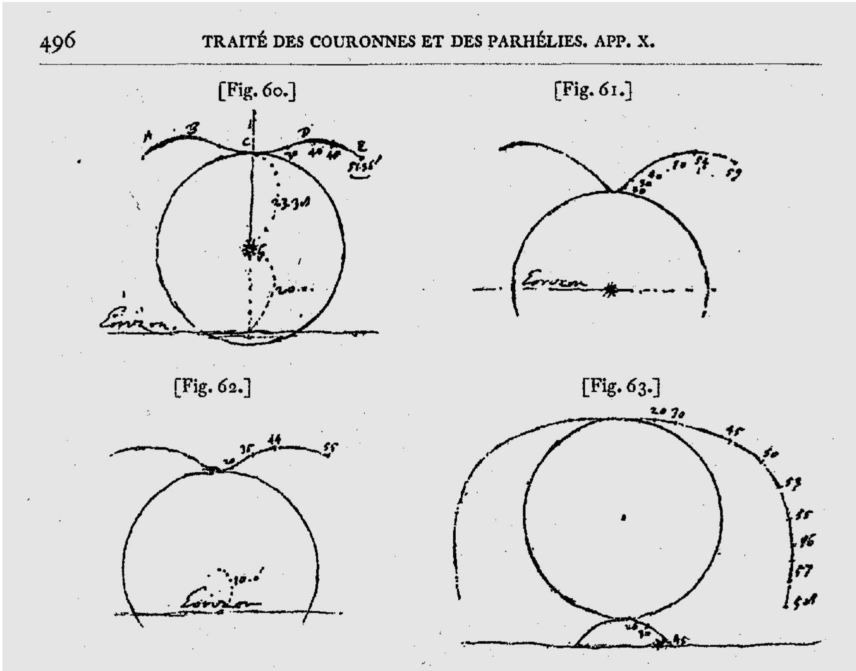


FIGURE 3.6 Part of a page from the collected works of Huygens [33], showing the essentially correct theoretical dependence of the tangent arc on sun elevation Σ. Compare the upper left diagram with the much later diagram of Wegener in Figure 5.1 or with the real tangent arc in Figure 1.8. Here Σ = 20°, 0°, 30°, 10°, clockwise from upper left. The horizon is labeled in script.

Hexagonal columns are common in cold climates, where light snowfall sometimes consists predominantly of such columns. They can be big enough to see—sometimes a millimeter or so in length—but not big enough to see well, not without a decent microscope. In the few cases where they were seen prior to the nineteenth century, the columns were usually taken to be cylindrical rather than prismatic. The first unequivocal reports of hexagonal columns seem to be those of Scoresby, in his *Account of the Arctic Regions* [66], which appeared in 1820.

In the eighteenth century the direct observational evidence for Mariotte's particles was therefore about the same as that for Huygens'—virtually nil—and the choice between Mariotte's theory and Huygens' theory was not so clear. In fact, the edge might logically have gone to Huygens, as indeed it did, since he had accounted for many more halos than had Mariotte, and since he had worked out the implications of his theory in more detail. Huygens, in fact, was far ahead of his time. He knew, for example, the fundamental result of classical halo theory later known as Bravais' law, and he had used it to calculate the appearance of the parhelia and the tangent arc as a function of sun elevation, all of this nearly two centuries before Bravais (Figure 3.6).

FIGURE 3.7 Beautiful lunar halo display, Jämsä, Finland, November 22, 2004. Compare the lower tangent arc here with Huygens' diagram at the lower right in Figure 3.6. Photo © Arto Oksanen.

Mariotte himself had treated only the 22° halo and the parhelia, but near the beginning of the nineteenth century his ideas were extended to explain other common halos. Young [91] in 1807 explained the tangent arc, and Young together with Cavendish explained the 46° halo. In 1840 G. Galle [18] explained the circumzenith arc.[3] Each of these explanations attributed the halos to hexagonal prismatic crystals having suitable orientations.

By this time both Huygens' theory and Mariotte's theory therefore offered explanations for most of the common halos: the 22° and 46° halos, the parhelia, the tangent arc, and the circumzenith arc. For these halos the predictions of the two theories are not much different. The predicted intensity distributions differ from one theory to the other, but the critical location of the inner, i.e., sunward,

[3]Galle's explanation was not correct. Galle, and indeed Huygens, confused the circumzenith arc with the upper tangent arc of the 46° halo, a largely hypothetical arc sometimes known as Galle's halo. (In the terminology of later chapters this halo is one of the 46° Lowitz arcs.) We have included Galle's explanation, somewhat artificially, in order to emphasize the similarities between Mariotte and Huygens. By 1840, when Galle's work appeared, Huygens' theory had already fallen from favor, even without having to face a tenable competing explanation of the circumzenith arc from Mariotte's side.

edge of each halo, where the halo tends to be brightest, turns out to be exactly the same for the two theories.

So in the early nineteenth century the choice between Huygens and Mariotte must have hinged mainly on the plausibility of the two ice particle models—hexagonal prisms versus layered spheres and cylinders—and there was still little direct observational evidence for either model. In the eighteenth century, when Mariotte's theory was incompletely worked out and therefore offered less competition, the difficulty of conceiving Huygens' spheres and cylinders with their ad hoc m-values probably would have been overlooked. But as Mariotte's theory was fleshed out, Huygens' spheres and cylinders must have seemed increasingly contrived, with the concocted m-values of 0.48 and 0.68 demanding explanation. By contrast, in Mariotte's theory the role of these two parameters is played by the natural interfacial angles of 60° and 90° on the hexagonal prisms, and no apologies are needed.

When one attempts to extend the theories of Huygens and Mariotte to the less common halos—most of which were little known and therefore not relevant at the time—Huygens' particle model fails badly and Mariotte's succeeds impressively, as we will see.

But you needn't take our word for it regarding the cause of halos, at least not if you live in a cold enough place. The north central United States will do, and so will northern Scandinavia. Just wait for one of those clear cold winter mornings when the air is filled with ice crystals. The crystals will be obvious, sparkling in the sunlight as they fall, but the sparkles may be more evident in some directions than in others. Those directions are the directions of halos forming in low level crystals, more or less in front of your nose. The halos can sometimes be bright enough to appear in front of nearby buildings or other objects (Figure 3.8). It will be obvious that halos have something to do with ice crystals and sunlight.

But what sorts of crystals are they? Again you can see for yourself, but you will need a microscope or strong hand lens unless the crystals are unusually large. Just expose a cold glass slide or dish for a few minutes to gather a sample of the falling crystals, and then take a look. If you were seeing good halos in your sparkles, you will normally find some hexagonal prisms—plates or columns—in your sample. For those of us living in cold places it is easy to believe that halos arise in prismatic ice crystals. Mariotte was right.

Although Mariotte was right about the responsible ice particles, it was Huygens who better understood the role of the orientation of the particles, and it was Huygens who had worked out the mathematics of the refraction of light rays in three dimensions. The theory of halos that developed at the beginning of the nineteenth century, and which forms the core of the modern theory, was a melding of the ideas of Mariotte and Huygens.

The failed ice particle model of Huygens is a nice reminder that much of science is an exercise in the logical fallacy known as affirming the consequent. (From the statements $p \rightarrow q$ and q, you infer p.) We are not complaining or criticizing; this is just the way science works. As an illustration, let's try to put ourselves again in eighteenth century shoes: We probably have not had a good look at an ice particle through a microscope, and we probably have never heard of Mariotte's theory. Who among us could look at Huygens' very explicit predictions for the tangent arc as a function of sun elevation (Figure 3.6), then see those same distinctive and unintuitive shapes confirmed in real tangent arcs (e.g., Figures 1.8 or 3.7), and then doubt Huygens' particle model? And of course it was not just the tangent arc; Huygens had made correct predictions for the other common halos as well. You may protest that you see no way for Huygens' particles to arise. Huygens did try to explain the origin of his particles (not very convincingly), but we suspect that he might also have responded much as did Alfred Wegener, who, when criticized for having no mechanism to explain his proposed continental drift, replied to the effect that, never mind, a mechanism will be found [82]. The point is that when the predictions resulting from a model conform so well to reality, that is, when "the consequent is affirmed," you tend to believe the model. As Huygens put it,

> ...at last I hit upon their [the halos'] true cause... For I make no scruple to call those causes true, whose effects agree so exactly with observations, as to make it seem unnecessary to search for others. (Smith's *Opticks* [69, page 200])

But there is always a tiny chance that the model is wrong, even if a great many of its consequences have been confirmed. And that was the case with Huygens' ice particle model.

Anyone interested in learning more about the early theory of halos will enjoy reading Auguste Bravais' 1847 *Mémoire sur les Halos...* [9]. For many years the *Mémoire* was quite literally the last word on halo theory, with virtually no important halo publications appearing again until the beginning of the twentieth century. Today, a century and a half after its publication, the *Mémoire* remains a classic reference.

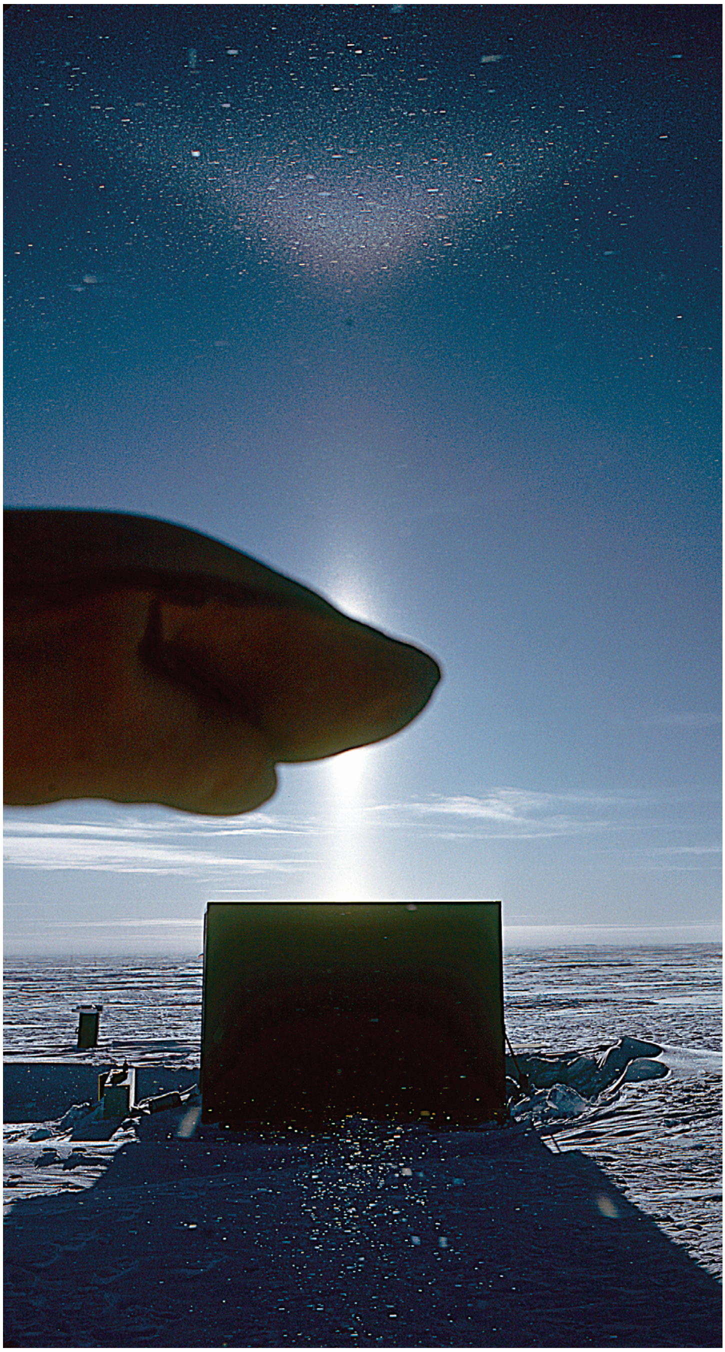


FIGURE 3.8 (*Opposite*) Evidence that halos are caused by ice crystals. In this low level halo display, discrete sparkles from nearby crystals are seen in the directions of the upper and lower tangent arcs. The lower tangent arc is showing up only as a concentration of sparkles between the photographer and the hut. South Pole, February 16, 1986.

FIGURE 3.9 More evidence that halos are caused by ice crystals. Here the 22° halo is forming in ice crystals on the grass. Eau Claire, Wisconsin.