CHAPTER 4

How Halos Form

Halos form when sunlight falls on suitable ice crystals in the atmosphere. But how exactly does it happen?

The sun rays are parallel

In general, it can be difficult to see why a given halo looks the way it does, and we will be grappling with this problem as we go along. The problem is simplified if we assume that all light rays from the sun are parallel. The sun therefore appears as a point in the sky and does not change its direction as an observer changes position. These assumptions are not far off; the angular diameter of the sun is indeed small, and the sun is far away. Actually the first assumption—that the sun appears as a point—is not crucial, and we can easily cope with a sun having positive angular diameter. We nevertheless assume, for the moment, that all sun rays are parallel. The sky is filled with them.

Light rays and light points

When thinking about halos, we often care as much about the light point of a light ray as we do about the ray itself. The *light point* of the ray is the point on the celestial sphere in the direction opposite to the ray. It is the point that appears to be lit, should the ray reach the eye of an observer.

The simplest halo display

What if the sky could somehow be filled with crystals all having exactly the same shape and same orientation? Suppose, for example, the crystals were all shaped

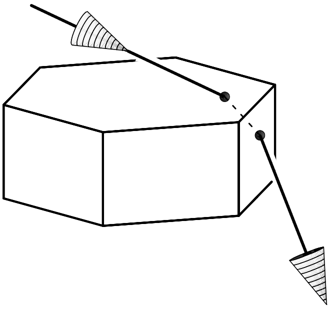

FIGURE 4.1 A common ray path through an oriented plate crystal. The ray from the sun enters the top basal face of the crystal and exits a prism face. This ray path contributes to the circumzenith arc. The arrowheads are drawn as solid cones so as to convey the directions of the rays in space; compare with Figure 4.2.

and oriented as in Figure 4.1. What would an observer see in the sky?

To answer such questions, you always need to think about the way light rays from the sun pass through the crystals. Consider, for example, a ray path as in Figure 4.1. The ray from the sun enters the upper basal face of the crystal and exits a prism face. The ray is refracted downward as it enters the crystal and is refracted once more as it exits.

The chance that the outgoing ray from this particular crystal reaches the eye of our fixed observer is nil. But throughout the sky the sun rays are parallel and the crystals are identical, so somewhere there will be a crystal having a ray path whose outgoing ray does reach the observer, and whose ray path is parallel, segment for segment, with the original ray path (Figure 4.2, upper right). The observer perceives a point of light in the direction opposite to that of the outgoing ray; this point is the light point of the outgoing ray, also known as the *halo point.* The halo display due to rays like those in Figure 4.1, passing through these identically shaped and oriented crystals, would consist of this single spot of light in the sky (Figure 4.2, bottom left).

Thus, in thinking about how halos form, you need only consider a single crystal, not a whole sky full of crystals. So long as every small region of sky is equivalent to every other, you need not worry about where the observer is nor whether the outgoing ray from the crystal actually reaches the observer. This is a big simplification.

A more realistic halo display

Let's now take the same crystal and rotate it about its axis, with the axis remaining vertical. As the crystal rotates slightly, the path of the outgoing ray changes slightly, and the halo point in the sky changes slightly as well. As the crystal continues to rotate, the halo point traces out a halo.

This halo happens to be the circumzenith arc. The responsible crystals are oriented plates, that is, plate crystals with their axes vertical. The responsible light

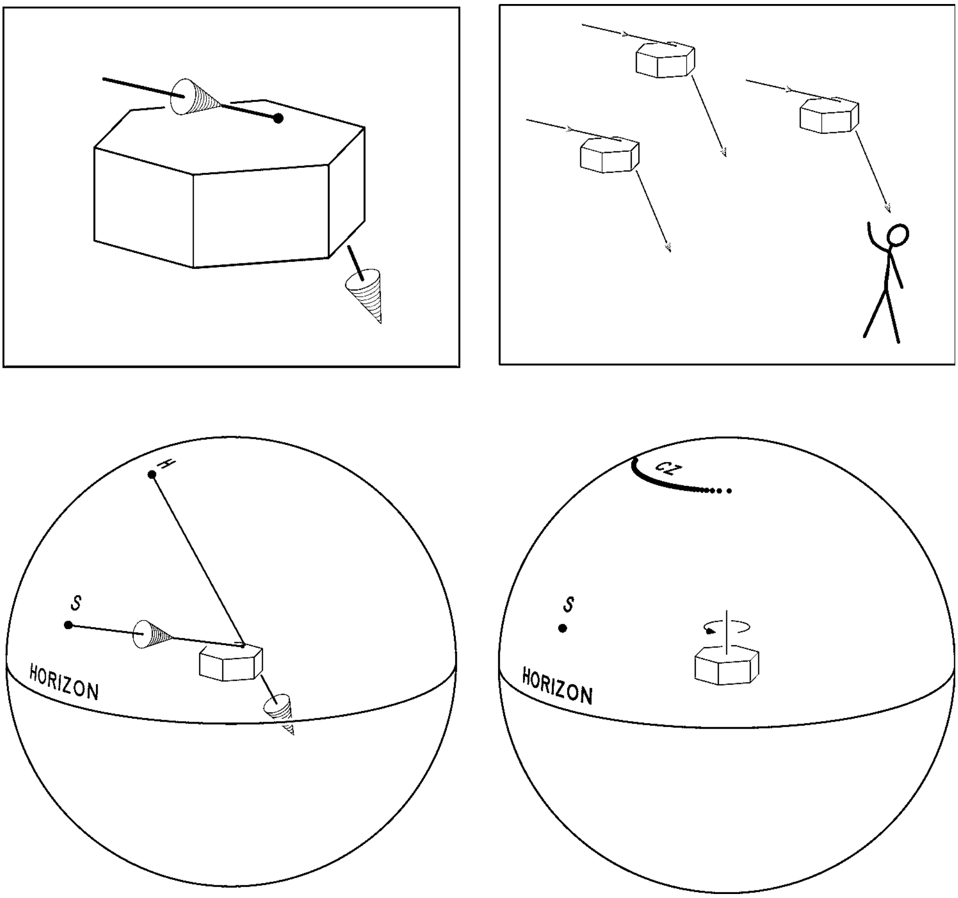

FIGURE 4.2 (*Top left*) Same crystal and ray path as in Figure 4.1 but seen from a slightly different viewpoint, one for which the halos can be more easily depicted on the celestial sphere. (*Top right*) Sky filled with crystals having shapes and orientations the same as at top left. An outgoing ray from one of the crystals reaches the observer. (*Bottom left*) Sun point **S** and halo point **H** on the celestial sphere. The halo point is the light point of the outgoing ray, that is, the point of the celestial sphere that appears lit to the observer. The crystal is at the center of the sphere. (*Bottom right*) Same as bottom left except that the crystal rotates about its (vertical) axis. As the crystal rotates, the halo point traces out a halo, the circumzenith arc **CZ**.

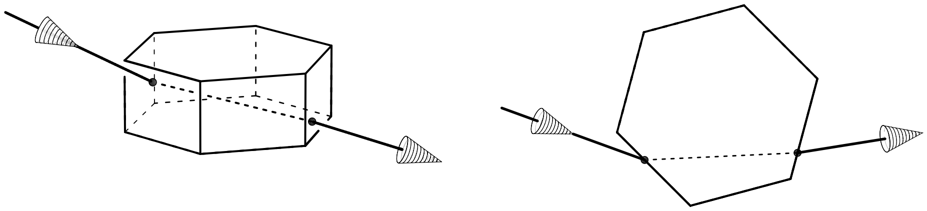

FIGURE 4.3 Another ray path through the oriented plate crystal of Figure 4.1. The ray enters a prism face and exits an alternate prism face. This ray path contributes to the left parhelion. (*Left*) The same perspective as in Figure 4.1. (*Right*) Top view of the same crystal and ray path.

rays are those that enter a top basal face and exit a prism face, as in Figure 4.1. It is clear from the figure, in which the ray is deviated sharply downward, why the circumzenith arc appears nearly overhead; this is the direction that an observer must look in order to see where the outgoing ray appears to be coming from.

The first halo display that we considered, consisting of a single spot of light, is of course entirely imaginary; you will never get the necessary sky full of identically shaped and oriented crystals. But the second display, arising in the rotating crystals, is another matter. Although you cannot expect a sky full of synchronized rotating crystals, you can expect something equivalent, namely, a sky full of oriented plate crystals. That is, each crystal should have its axis (more or less) vertical, and all rotational orientations about the axis should be equally likely. Individual crystals need not be rotating.

Different ray paths make different halos

Let’s return now to the first halo display, the imaginary display produced by the identically shaped and oriented crystals. The ray path that we considered (Figure 4.1) is not the only way that a sun ray might pass through the crystal. A ray might, for example, enter a prism face and exit another prism face as in Figure 4.3. The new outgoing ray would have an entirely different direction from that of the outgoing ray for the circumzenith arc, and hence it would have an entirely different halo point on the celestial sphere. We see that the halo display would actually consist of at least two points of light, not just one.

If, as before, we let the crystal rotate about its axis, then the new halo point traces out a new halo, which happens to be the left parhelion (left, as seen from inside the celestial sphere). The two different ray paths cause two different halos.

Still other ray paths through the same oriented plate crystal are possible. Perhaps a dozen of them have outgoing rays that are bright enough to be significant. The first display that we considered, with a fixed crystal orientation, actually consists of about a dozen isolated spots of light in the sky; these are halo points of various outgoing rays. In the second display, where the crystal is allowed to rotate about its axis, each halo point traces out a halo, thus giving about a dozen halos. In general, a halo is the locus of the halo point of a specified ray path as a crystal takes on orientations in the given orientation class. Change the ray path and you (usually) get a new halo.

Both displays are shown in Figure 4.4. In making the right hand diagram, we only allowed the crystal to rotate through an angle of 15° (in increments of one degree), so as to keep the diagram from becoming impossibly busy. Most of the halos are therefore incomplete. The parhelic circle, for instance, if complete, would be a circle going all the way around the sky at the elevation of the sun.

In the figure each ray path is written as a sequence of numbers, namely, the numbers of the crystal faces that the ray encounters. The basal faces of the crystal are numbered 1 and 2, and the prism faces are numbered 3, 4, 5, 6, 7, 8, in counterclockwise sequence when looking at face 1. The face numbers are shown in Figure 4.4 and more clearly in Figure 6.1. But the details of the ray paths are not so important at the moment. The main point here is that different ray paths give different halos.

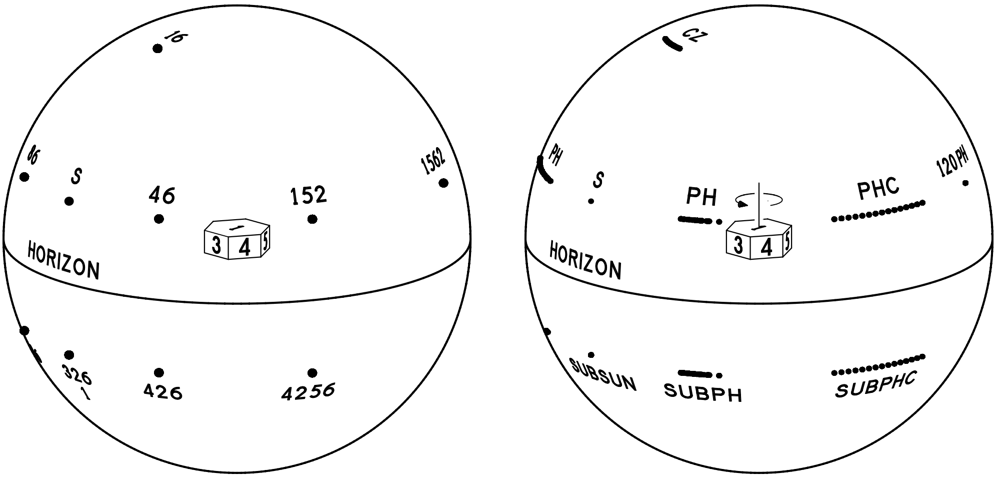

FIGURE 4.4 Same as the bottom diagrams of Figure 4.2 but with halo points from additional ray paths. (*Left*) The crystal orientation is fixed, the same as in the bottom left diagram of Figure 4.2. The resulting imaginary halo display consists of about a dozen isolated halo points. Each halo point is labeled with its ray path. (*Right*) The crystal is allowed to rotate. Each halo point in the left diagram begins to trace out a halo. **CZ** = circumzenith arc, **PH** = parhelion, **PHC** = parhelic circle, **120 PH** = 120° parhelion, **SUBPH** = subparhelion, **SUBPHC** = subparhelic circle.

There are exceptions. According to Figure 4.4, the two ray paths 1 (reflect off the top basal face) and 3 2 6 (enter a prism face, reflect off the bottom basal face, and exit the prism face opposite the entry face) both contribute to the halo known as the subsun. This is clear if you can see that the direction of the outgoing ray is the same for both ray paths. Another example: The ray paths 3 5 and 4 6 both contribute to the left parhelion. But here the distinction between the two paths is superficial, being essentially an accident of the face numbering.

The point remains that different ray paths usually do make different halos. The existence of many essentially different ray paths is one factor that makes for a rich variety of halos.

Why the halos look the way they do

This chapter was supposed to tell how halos form. Perhaps it did, but it did not explain why the halos look the way they do. To do so, we need to be able to find the paths of the light rays through the crystals, so that we can in turn find the halo points on the celestial sphere. What is needed are the laws of reflection and refraction, but in a geometrical, conceptual form.

In order to express these laws simply, we revise slightly our notion of light point. The light point of a ray will still be the point on the celestial sphere that the ray appears to light, but the celestial sphere will now be regarded as having radius equal to the refractive index of the medium containing the ray. Thus there must be a separate celestial sphere for each refractive index. All of them are concentric.

Laws of reflection and refraction *Suppose a light ray is incident on a boundary between two media. Then the light points* **S**, **S′**, *and* **R** *of the incident, reflected, and refracted rays all lie on a line normal to the boundary (Figure 4.5).*

Thus to get **S′** and **R** from **S**, you just project **S** in a direction normal to the boundary; the point **S′** is on the same sphere as **S** but on the opposite side of the boundary, and the point **R** is on the other sphere but on the same side of the boundary.

Consider as an example the ray path 1 in oriented plate crystals; it is just a reflection off the top basal face of the crystal (Figure 4.6). What is the resulting halo? The incoming and outgoing rays are both external to the crystal, and so their light points—the sun point **S** and the halo point **H**—are both on the sphere of radius 1 (refractive index of air). The boundary between the two media (air and ice) is the top face of the crystal, which is horizontal. The normal direction is therefore vertical, and so **H** is directly below **S**, at the so-called subhelic point. The halo is the subsun.

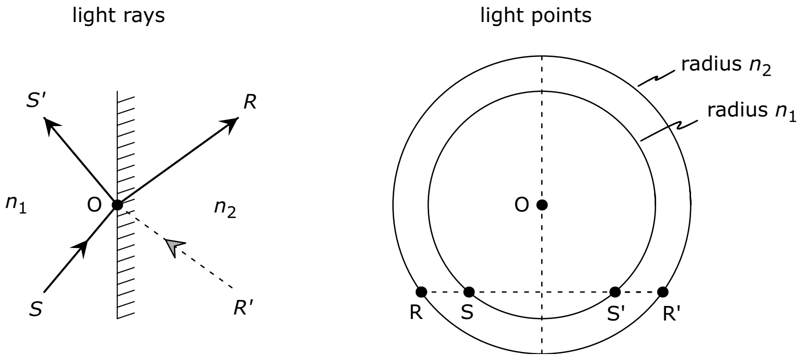

FIGURE 4.5 How to find refracted and reflected rays. (*Left*) Incident, reflected, and refracted rays *S*, *S′*, and *R* at a boundary between media having refractive indices n_1 and n_2. The boundary is perpendicular to the plane of the paper, and all rays are in the plane of the paper. The dashed ray *R′* is just to complete the symmetry; it is the ray that would reflect to *R* and refract to *S′*. (*Right*) The corresponding light points **S**, **S′**, **R** and **R′**. The light points all lie on a line normal to the boundary, and any one of them therefore determines the others.

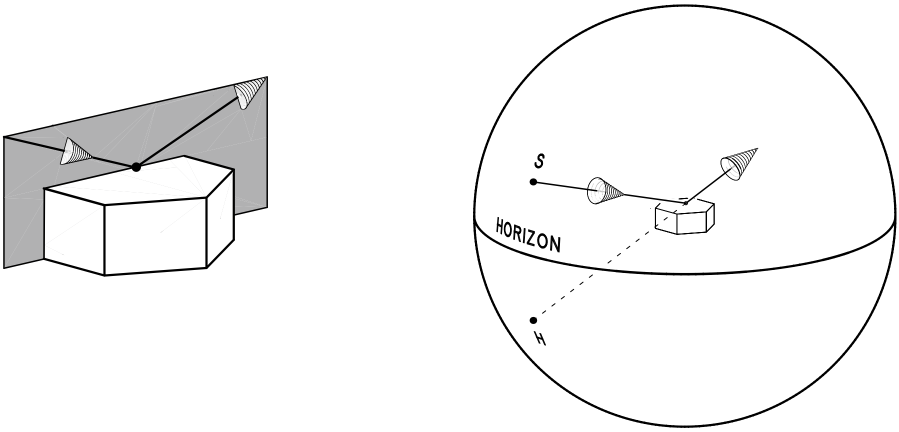

FIGURE 4.6 Formation of the subsun by a ray reflecting off the top face of an oriented plate crystal. According to the law of reflection, the halo point **H** must be on the same sphere as the sun point **S** and directly below it, since the normal to the reflecting face is vertical. The right-hand diagram is the "light point diagram" for the subsun. The crystal is at the center of the sphere.

The subsun is a common halo, perhaps the most common halo, but how many people have seen it? The problem in seeing it, is that you need to be above the ice crystals. Either you need to be in a place cold enough that atmospheric ice crystals can occur near the ground, or you need to fly. But whenever you can manage to have sunlit crystals below you, look for the subsun. It is just a bright spot below the sun, exactly as far below the horizon as the sun is above it.

For a second illustration of the laws of reflection and refraction, refer to Figure 4.7, which shows how the circumzenith arc is formed. Now two spheres are

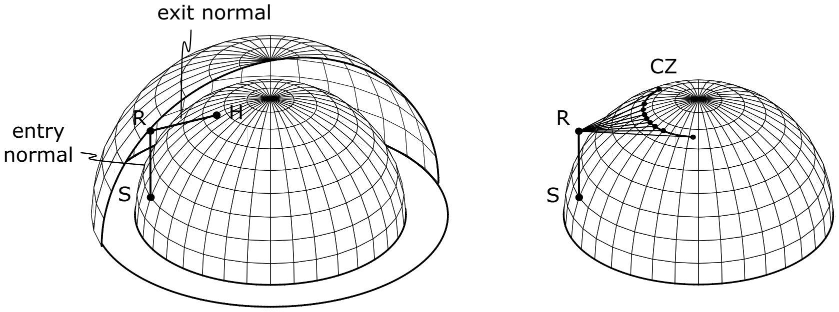

FIGURE 4.7 Light point diagram for the circumzenith arc, showing the formation of the arc. (*Left*) Light points **S**, **R**, and **H** of the entry, internal, and exit rays for the ray path of Figure 4.1. The halo point **H** is found by projecting the sun point **S** to the outer sphere, thus getting **R**, and then projecting **R** to the inner sphere, getting **H.** See text. (*Right*) Same but for many different rotational orientations of the crystal about the vertical. The halo point traces out the circumzenith arc **CZ.** The spheres have radii 1 and 1.31, the refractive indices of air and ice.

required—an inner sphere of radius 1, to accommodate light points of external rays, and an outer sphere of radius 1.31 (refractive index of ice) to accommodate light points of internal rays. For a given crystal orientation the halo point is found by first projecting the sun point to the outer sphere, then projecting the resulting point back to the inner sphere. Each projection direction is normal to the relevant crystal face; the first projection is vertical, since the entry face is horizontal, and the second projection is horizontal, since the exit face is vertical (Figure 4.1). From Figure 4.7 it is easy to see why the circumzenith arc is part of a horizontal circle; the arc must lie on the intersection of a horizontal plane with the celestial sphere.

Figures 4.6 and 4.7 are "light point diagrams." They show how to construct the halos, starting with the crystal shapes and orientations and using the laws of reflection and refraction. If you understood those figures, you may enjoy thinking about the light point diagrams for other ray paths or other crystal orientations. Try, for example, the ray path 3 2 6 in oriented plate crystals; it contributes to the subsun. Or try the ray paths 1 or 3 2 6 in oriented column crystals; they contribute to the parhelic circle.

Light point diagrams for a few more halos appear in Chapter 6. For many other halos, however, we will have to rely on simulations or on "pole theory" to understand them, rather than on their light point diagrams. It is not that their light point diagrams fail to be correct. Rather, they become too complex and not very enlightening. The diagrams can be drawn by computer, and they are accurate, but they can still be hard to make sense of, sometimes just a jumble

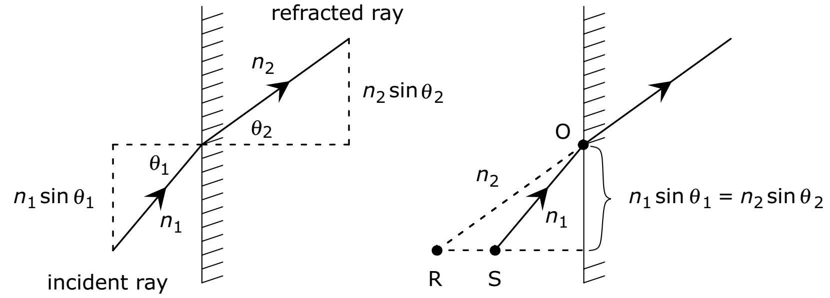

FIGURE 4.8 Derivation of the geometric version of the law of refraction. (*Left*) Incident and refracted rays as in Figure 4.5, drawn with lengths of n_1 and n_2, respectively. The traditional law of refraction, namely, $n_1 \sin\theta_1 = n_2 \sin\theta_2$, ensures that the tangential components of the two rays are equal. (*Right*) Same but showing the light points **S** and **R** of the incident and refracted rays. The line segment **SR** is normal to the boundary, as claimed. The celestial spheres, not shown here, are centered at the point **O**.

of lines. Even a halo as common and relatively simple as the tangent arc turns out to have a fairly complex light point diagram. So after Chapter 8 you will not see any more light point diagrams.[1] But the fundamental laws of reflection and refraction will always be there, incorporated somewhere in the theory.

The geometrical versions of the laws of reflection and refraction that we have been using are not as widely known as they deserve to be, though they date back at least to Hamilton [23] and MacCullagh [44] in the nineteenth century. The geometric version of the law of refraction is derived in Figure 4.8.

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[1] Not all the halos of later chapters are complicated. The upper 23° plate arc (Chapter 15) would be an example, its light point diagram being similar to that of Figure 6.9.