## Halos From Prismatic Crystals

This chapter treats what might be called the classical halos-the halos arising in the simplest sorts of ice crystals, namely, hexagonal prismatic crystals. These halos, then, are not the odd radius halos. The treatment is not intended to be comprehensive. More information, including ray path diagrams and many more simulations, can be found in the book Atmospheric Halos [76]. Also, at the time of this writing there was at least one halo simulation program available on the internet. ${ }^{1}$ With it, you can change the sun elevation, the crystal shapes, crystal orientations, etc., and see for yourself how the halos are expected to change.

The halos in this chapter are organized according to the crystal orientations that make them (Figure 6.1). By far the most common orientations are plate orientations, column orientations, and random orientations. Parry orientations are rare, and Lowitz orientations are rarer still.

## Plate arcs

Recall that a crystal is in plate orientation when its two basal faces are (more or less) horizontal. Numbering the crystal faces $1,2, \ldots, 8$ as in Figure 6.1, we always assume-without loss of generality - that in plate orientation it is face 1 that is the top horizontal face. The halos that arise in crystals with plate orientations are known as plate arcs. Plate arcs from prismatic crystals are shown in Figure 6.2. The $22^{\circ}$ plate arcs are the plate arcs that have the same ray path as the $22^{\circ}$ circular halo, and the $46^{\circ}$ plate arcs are those that have the same ray path as the $46^{\circ}$ circular halo.

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FIGURE 6.1 (Far left) Hexagonal prismatic crystal with standard face numbering. The basal faces are numbered 1 and 2 , and the prism faces are $3,4,5,6,7,8$, in counterclockwise sequence when looking directly at face 1. (Center left) Same crystal but with plate orientation; face 1 is horizontal and on top. (Center) Same but with Parry orientation; face 3 is horizontal and on top. (Center right) Same but with column orientation; faces 1 and 2 are vertical. (Far right) Same but with Lowitz orientation. The indicated axis remains horizontal.

Plate arcs can be understood using the laws of reflection and refraction (page 38), as was done for the subsun and circumzenith arc in Figures 4.6 and 4.7. The parhelic circle and the circumhorizon arc are easy, the parhelia and the $120^{\circ}$ parhelia require more thought.

You will find that plate arcs from prismatic crystals are confined to at most four horizontal circles on the celestial sphere. For the sun elevation of Figure 6.3 the circles $C_{1}$ and $C_{2}$ - the parhelic and subparhelic circles-would contain the parhelia and subparhelia, respectively, and the circles $C_{3}$ and $C_{4}$ would contain the circumzenith arc and the (not yet observed) subcircumzenith arc. Plate arcs lie on these four circles because all of the projection directions called for by the laws of reflection and refraction are either horizontal or vertical.

## Column arcs

Recall that a crystal is in column orientation when the crystal axis is horizontal. The halos that arise in crystals with column orientations are known as column arcs. Column arcs from prismatic crystals are shown in Figure 6.4.

In principle you can understand most column arcs if you understand plate arcs. We now describe briefly how this goes, but mainly for historical reasons; it is not something that we need later in the book. The idea is that if you fix the crystal axis and let the crystal spin about the axis, you get "rotated plate arcs." These arcs are like plate arcs but with the point on the celestial sphere in the direction of the crystal axis playing the role of the zenith. (Forget about the horizon, pretend that "up" is in the direction of the crystal axis.) This is the idea behind Wegener's depiction of the tangent arc (our Figure 5.1). There the two red segments are the "rotated parhelia" made by the ray paths 35 and 37 as the crystal spins about its axis. These rotated parhelia lie on a circle through the sun (not shown), but the circle is vertical, not horizontal, since its center is a point on the horizon - the point



FIGURE 6.2 All-sky simulations showing plate arcs from prismatic crystals. The crystals are shaped like the one shown. Sun elevations $\Sigma=20^{\circ}, 60^{\circ}$. The table gives common ray paths for the halos.



FIGURE 6.3 The four horizontal circles $C_{1}, C_{2^{\prime}}$ $C_{3^{\prime}} C_{4}$ (heavy segments) of the celestial sphere where plate arcs from prismatic crystals can occur. Here the spheres of radii 1 and $n=1.31$ are seen in cross section, with the sun at $\mathbf{S}$. The circles change with sun elevation, but there are always at most four.
in the direction of the crystal axis. (If you rotate the celestial sphere, including the sun, in such a way that the circle becomes horizontal, then the parhelia become bona fide parhelia.) Similarly, each of the curved segments in Wegener's tangent arc is a rotated parhelion. The parhelia come in pairs, one in the upper tangent arc and one in the lower, with each pair corresponding to a different fixed horizontal direction of the crystal axis. The apparent asymmetry between the two parhelia in each pair is only due to Wegener's choice of projection.

This insight-that column arcs consist of rotated plate arcs-goes all the way back to Huygens. He used it, as just described, to calculate the correct shape of the tangent arc (Figure 3.6), even though his ice particle model was not correct. Using this same insight, Bravais recognized that, just as the tangent arc is the column-analogue of the parhelia, the infralateral arc is the column-analogue of the circumzenith arc (both have ray path 13), and the supralateral arc is the columnanalogue of the circumhorizon arc (both have ray path 32). Later Wegener explained the halo now known as the Wegener arc; it is the column-analogue of the subparhelia (both have ray paths 325 and 327 ). Wegener also recognized that the subhelic arc is the column-analogue of the $120^{\circ}$ parhelia. Although the Huygens-Bravais-Wegener insight is the classical approach to understanding column arcs, some care is required to carry it out in detail, as can be seen even in Wegener's diagram.

For column orientations the helic, subhelic, anthelic, and subanthelic points of the celestial sphere play a special role (Figure 6.5). To see why, imagine again that the crystal axis is fixed (and horizontal) and that the crystal rotates about the axis. The resulting "halos" lie on four vertical circles, just as the halos from plate orientations lie on four horizontal circles. Two of those vertical circles are relevant at the moment. One passes through the helic point (the sun) and the subhelic point, and the other passes through the anthelic point and the subanthelic point. This is true regardless of the (horizontal) direction of the crystal axis,


| Halo | Ray path |  |  |
| :---: | :--- | :--- | :--- |
| $\mathbf{2 2 ^ { \circ }}$ column arcs | Tangent arc | T | 35 |
| $\mathbf{4 6 ^ { \circ }}$ column arcs | $\left\{\begin{array}{lll}\text { Infralateral arc } & \text { INF } & 13 \\ \text { Supralateral arc } & \text { SUP } & 31\end{array}\right.$ |  |  |
| Arcs involving |  |  |  |
| reflection |  |  |  |\(\left\{\begin{array}{lll}Parhelic circle \& PHC \& 1,316 <br>

Wegener arc \& W \& 315 <br>
Subhelic arc \& \mathbf{H}_{2} \& 1342,1352 <br>
Anthelic arc \& \& 31574,12351 <br>
Diffuse arcs \& D \& 3157,325673\end{array}\right.\)

FIGURE 6.4 Column arcs from prismatic crystals. The anthelic arc is listed in the table but appears in neither simulation.



FIGURE 6.5 Helic, subhelic, anthelic, and subanthelic points on the celestial sphere.
even though the circles themselves do depend on the direction of the crystal axis. Column orientations therefore have a natural potential to focus light on the helic, subhelic, anthelic, and subanthelic points; near these points many different crystal orientations can work together to light the sky. Whether you actually get light at these points, however, depends on a number of other factors as well.

## Arcs from Parry orientations

A crystal is in Parry orientation when two prism faces are horizontal. We always assume that in Parry orientation it is face 3 that is the top horizontal face. Crystals with Parry orientations are more severely constrained than are those with column orientations. Whereas a crystal in column orientation is free to rotate both about its own axis and about the vertical axis, a crystal in Parry orientation can only rotate about the vertical axis.

Arcs from Parry orientations are shown in the simulations in Figure 6.6. Although these arcs are rare, they do occur, and they can be spectacular, as in the display shown in Figures 6.7 and 6.8. Most of the halos identified in the $\Sigma=20^{\circ}$ simulation can be seen in that display. Arcs from Parry orientations can also be seen in Figure 1.8; we ignored them earlier, in the interests of simplicity.

Arcs from Parry orientations are easier to understand than are column arcs (i.e., halos from column orientations), since, as for plate arcs, the set of responsible crystal orientations is one-dimensional rather than two-dimensional.

Figure 6.9 shows how the upper suncave ( $22^{\circ}$ ) Parry arc forms. The responsible ray path is 35 ; the ray enters the top horizontal prism face and exits a lower sloping prism face. The normal direction to the entry face is therefore vertical, just as it was for the circumzenith arc (Figure 4.7). Therefore, to get the light point $\mathbf{R}$ of the internal ray, you project the sun point directly upward to the outer sphere. Then to get the halo point, you project $\mathbf{R}$ back to the inner sphere, with the projection being in the direction normal to the exit face. The normal


| Halo |  |  | Ray path35 |
| :---: | :---: | :---: | :---: |
| $22^{\circ}$ Parry arcs | Upper suncave | $\mathrm{P}_{1}$ |  |
|  | Lower suncave | $\mathrm{P}_{2}$ | 46 |
|  | Upper sunvex |  | 48 |
|  | Lower sunvex |  | 57 |
| $46^{\circ}$ Parry arcs | Circumzenith arc | CZ | 31 |
|  | Circumhorizon arc | CH | 16 |
|  | Infralateral | 146 | 17, 15 |
|  | Supralateral | S46 | 41, 81 |
| Arcs involving reflection | Parhelic circle | PHC | 1,417 |
|  | Hastings arc (of $\mathrm{P}_{1}$ ) | $\overline{P_{1}}$ | 315 |
|  | Hastings arc (of $\mathbf{P}_{2}$ ) | $\overline{P_{2}}$ | 416 |
|  | Helic arc | $\mathrm{H}_{1}$ | 4, 5, 152, 345 |
|  | Subhelic arc | $\mathrm{H}_{2}$ | 1452 |
|  | Anthelic arc |  | 36184,418765485 |
|  | Subanthelic arc | $\mathrm{H}_{4}$ | 3145,4187654386 |

FIGURE 6.6 Arcs from prismatic crystals having Parry orientations. Not all of the arcs in the table appear in the simulations.



FIGURE 6.7 Halo display, South Pole, January 11, 1999. The display has halos from plate, column, Parry, and random crystal orientations; nearly all of the labeled halos in the $\Sigma=20^{\circ}$ simulations of Figures $6.2,6.4$, and 6.6 appear here. In this display the halos from Parry orientations are the best that we know of. Sun elevation $\Sigma=22^{\circ}$. Photo © Marko Riikonen.


FIGURE 6.8 Same display as in Figure 6.7. Arcs here that arise exclusively from Parry orientations are the upper suncave Parry arc, the left supralateral $\left(46^{\circ}\right)$ Parry arc, the helic arc, the subanthelic arc, and the Hastings arc, the latter perhaps being too faint to survive reproduction.


FIGURE 6.9 Light point diagram for the upper suncave Parry arc, showing the formation of the arc. The right-hand diagram emphasizes that the halo is on the intersection of a cone with the celestial sphere. Compare with Figure 4.7.

light points of internal rays
FIGURE 6.10 Light point diagram for the upper sunvex Parry arc. The light points of the internal rays are on the outer sphere as usual, but only the inner sphere is shown here.
to the exit face, instead of being horizontal as it was for the circumzenith arc, is now inclined $30^{\circ}$ to the horizontal. Thus the segments emanating from $\mathbf{R}$ in the figure make up part of a cone. The halo is on the intersection of the cone with the celestial sphere.

Figure 6.10 shows the formation of the upper sunvex Parry arc. The ray path is 48 ; the ray enters and exits the upper sloping prism faces. As usual, you get the halo point by projecting the sun point $\mathbf{S}$ to the outer sphere and then back to the inner sphere. For each crystal orientation the two projection directions are inclined $30^{\circ}$ to the horizontal and are in a vertical plane passing through $\mathbf{S}$.

It is clear from the previous example - the upper sunvex Parry arc-that the circumzenith arc (Figure 4.7) and the upper suncave Parry arc (Figure 6.9) owe their simplicity to a kind of degeneracy, and that they are not representative of halos in general. But even the upper sunvex Parry arc has special properties that

helic arc

helic and subanthelic arcs

FIGURE 6.11 (Left) Light point diagram for the helic arc. The arc is the intersection of the celestial sphere with a cone whose vertex is at the sun. (Right) Same but also showing the subanthelic arc, which is the intersection of the sphere with a cone whose vertex is at the subanthelic point.
make it relatively simple and not entirely representative. Fortunately for the early halo theorists, most of the classical halos are special in one way or another, and they are therefore simpler to understand than they might have been. Of the halos arising in Parry orientations, the ones that are least special are the infralateral and supralateral $46^{\circ}$ Parry arcs ( $\mathbf{1 4 6}$ and $\mathbf{s} 46$ in Figure 6.6), but they were virtually unknown until recently. Although it is easy enough to instruct the computer to draw the analogues of Figure 6.9 for these arcs, the resulting diagrams are not all that helpful, and we omit them.

Far simpler to understand are the four closely related helic, subhelic, anthelic (or Tricker), and subanthelic arcs. The simplest is the helic arc, and it is simple indeed (Figure 6.11). The responsible ray path is essentially a single reflection off a sloping prism face. Only a single sphere, of radius one, is required to depict the formation of the halo, and all of the projections go from this sphere to itself. Each halo point is found by projecting the sun point in the direction normal to the reflecting face. Since the crystal is in Parry orientation, the normal direction is inclined $30^{\circ}$ to the horizontal plane. As the crystal rotates about the vertical, the normal directions sweep out a vertical cone with vertex at the sun and with a vertex half-angle of $60^{\circ}$. The helic arc is the intersection of the cone with the celestial sphere.

The subanthelic arc, also shown in the figure, is the intersection of another cone with the celestial sphere, but it is harder to see why. The cone turns out to have vertex at the subanthelic point and to have a vertex half-angle of $30^{\circ}$. For the subhelic and anthelic arcs, you reflect this figure in the horizontal plane.


FIGURE 6.12 Arcs from prismatic crystals having Lowitz orientations. The $46^{\circ}$ Lowitz arcs with ray paths 14 and 15 are long arcs that are difficult to pick out in the simulation; they are not labeled. Sun elevation $\Sigma=20^{\circ}$. The crystal spins about the indicated horizontal axis.


## Arcs from Lowitz orientations

Arcs from Lowitz orientations are rare. We mention them here partly because Lowitz arcs are part of halo lore, and partly because having available this additional class of crystal orientations (Lowitz orientations) puts the other crystal orientation classes in better perspective.

A crystal is in Lowitz orientation when a long axis of a basal face is horizontal. Physically, the crystal is supposed to be spinning about this horizontal axis, or rather about a parallel axis through the center of the crystal.

Arcs from Lowitz orientations are shown in Figure 6.12. The nomenclature for these arcs is not settled, and in the simulation we have just labeled the arcs with their ray paths. The arc 46 is presumably the arc seen by Tobias Lowitz in the famous St. Petersburg display of 1790; Lowitz [43] described it as a short colored arc extending from the parhelion downward towards the $22^{\circ}$ halo. Lowitz did not, however, report any of the other arcs that might be expected from Lowitz orientations.

In the past we expressed doubts about the existence of Lowitz arcs. We were wrong, and in the past decade a number of photographs have been taken that show $22^{\circ}$ Lowitz arcs.

Some of these photos have been analyzed by Riikonen et al [60]. They found that while a simple spinning crystal like that of Figure 6.12 would simulate the shapes of the halos correctly, it failed to get the intensities right. To get reasonable intensities they had to adjust both the shapes and orientations of the crystals.

## The circular halos

More or less randomly oriented prismatic crystals make the $22^{\circ}$ and $46^{\circ}$ circular halos. The orientations need not be completely random, and in fact truly random orientations may be the exception rather than the rule. But to avoid repeated qualifications, we will nevertheless speak of the circular halos as arising in randomly oriented crystals. Circular halos are treated in detail in Chapter 8.

## Sun pillar

The sun pillar is a vertical column of light that seemingly emanates from the rising or setting sun (e.g., Figure 18.3, top). It is due to reflections of sunlight in nearly horizontal crystal faces. Most often the crystals are oriented hexagonal plates, but other shapes and orientations are also possible; all that is required is low sun and a preponderance of nearly horizontal crystal faces.

The sun pillar is anomalous in the following sense: Most (non-circular) halos are best when the tilts of the crystals are nearly zero. But the sun pillar disappears when the tilts are zero. (Or it becomes the subsun, depending on your point of view.) In this sense the sun pillar does not have a halo identity of its own.


[^0]:    ${ }^{1}$ http://www.sundog.clara.co.uk/halo/halosim.htm. This website has much more on halos, including many fine photographs.

