## A User's Guide To Halo Poles

The concept of halo pole is crucial for understanding the many odd radius halos that can arise in preferentially oriented crystals. In this chapter we will try to tell enough about the concept so that it can be used in practice, but we won't say much by way of justification until Appendix F. For more of the logic behind the concept of halo pole, you should refer to the article by Tape and Können [77]. It was Können who originated the concept, in the 1990s.

Except for the circular halos, almost every halo has a so-called pole. If the halo is a non-contact arc, such as a plate arc or Parry arc, then the pole and the wedge angle together completely determine the halo. In fact, for a given sun elevation the qualitative appearance of the halo can be immediately inferred from the pole and wedge angle, and it can be done with little or no calculation. It is remarkable. And the same is true if the halo is a contact arc, such as a column arc or Lowitz arc: the pole and the wedge angle determine the halo, though not so simply as for non-contact arcs. It is therefore worth the effort to learn how to find halo poles, not just for odd radius halos but for halos in general.

It is no secret: The halo pole is just the spin vector as seen from the halo-making wedge. But we need a couple of preliminaries.

Unit sphere The unit sphere is a standard sphere of radius one. We use it to represent directions in space; a point on the unit sphere represents the direction of an arrow drawn to the point from the center of the sphere, as shown in Figure 14.1. We often opt not to distinguish between the point and the arrow. Thus the letter $\mathbf{V}$ in the figure refers either to the point or to the corresponding arrow.

We place a standard cartesian coordinate system at the center of the sphere, with the $x$-axis pointing to the front, the $y$-axis to the right, and the $z$-axis up.


FIGURE 14.1 Unit sphere. Each point V on the sphere represents a direction in space, namely, the direction of an arrow drawn to the point from the center of the sphere.


FIGURE 14.2 Halo-making wedge in standard orientation. The entry face of the wedge is toward the front of the sphere, the exit face to the rear.

Then the north pole of the sphere, for example, is the point $(0,0,1)$, and the equator is in the plane $z=0$.

Wedge in standard orientation A halo-making wedge placed at the center of the sphere is said to be in standard orientation (Figure 14.2) when its entry and exit faces are oriented symmetrically with respect to the plane $x=0$ that separates the front hemisphere from the rear, and when the entry face is toward the front, the exit face is toward the rear, and the wedge opens downward.

## To find halo poles

The pole of a halo is a certain point on the unit sphere. The pole can be defined and calculated analytically, but we prefer here a less formal approach. As a first example we consider the circumzenith arc made by crystals having Parry orientations (Figure 6.6). The ray path for the halo enters face 3 (the top prism face) of the crystal and exits face 1 (a basal face). We recommend that you construct a model of the crystal, number the faces as usual, and insert a nail or dowel part way into face 3 to represent $\mathbf{N}_{3}$, the spin vector $\mathbf{P}$ for Parry orientations (Figure 13.1). Then to get the pole of the halo, you simply orient the crystal model so that wedge 31 is in standard orientation. When the wedge is in standard orientation the head of the nail (the spin vector $\mathbf{P}$ ) points in the direction of the pole of the halo. In this example the pole is therefore as shown in Figure 14.3; it is on the meridian $(y=0)$ of the sphere and $45^{\circ}$ above the equator, since the wedge angle is $90^{\circ}$.


FIGURE 14.3 The pole of the circumzenith arc that arises in Parry orientations. The halomaking wedge is wedge 31 , and the spin vector $\mathbf{P}$ is $\mathbf{N}_{3}$. When the wedge is in standard orientation, as here, the spin vector $\mathbf{P}$ points in the direction of the halo pole. The left-hand diagram shows the entire crystal, whereas the right-hand diagram, in order to focus attention on the halo-making wedge, shows only the wedge.


FIGURE 14.4 The pole of the halo that arises in Parry orientations and that has wedge 81 . See text. The halo is the upper right $46^{\circ}$ Parry arc.

For a second example of a halo pole, consider the halo that arises in crystals having Parry orientations and that has ray path 81 . The only difference between this example and the previous one is that the ray path is now 81 rather than 31. The spin vector $\mathbf{P}$ is therefore $\mathbf{N}_{3}$ as before, but now it is wedge 81 that must be put in standard orientation in order to find the pole of the halo. So start with your crystal model oriented as in Figure 14.3, with wedge 31 in standard orientation as before, and then rotate it $60^{\circ}$ about its $c$-axis. The wedge 81 moves into standard orientation, and the point $\mathbf{P}$ moves along the great circle shown in Figure 14.4, eventually arriving at the position shown there - the pole of the halo.

The preceding halos are two of several $46^{\circ}$ Parry arcs, that is, refraction arcs that have wedge angle $\alpha=90^{\circ}$ and that arise in crystals having Parry orientations.

The poles of the $46^{\circ}$ Parry arcs are shown in Figure 14.5. You can verify the pole locations using your crystal model as above; just treat each of the $90^{\circ}$ wedges in succession. You will find, for example, that the Parry arc with ray path 15 has the pole shown at the lower right of the sphere; that is the direction of the nail when your model is oriented so that wedge 15 is in standard orientation.

How would the procedure change if the halos were plate arcs instead of Parry arcs? Only in that the spin vector $\mathbf{P}$ would then be $\mathbf{N}_{1}$, the outward normal to face 1, instead of $\mathbf{N}_{3}$. So you would need to move the nail to face 1 of your crystal model. The pole of the right parhelion, for example, would turn out to be the point $(0,1,0)$ at the far right of the sphere.

Perhaps you can see from these examples that the pole of the halo is just a convenient means of specifying the wedge orientations that are responsible for the halo. That is why the pole, together with the wedge angle, determines the halo.

## To infer the appearance of a non-contact arc from its pole

We consider as examples the $46^{\circ}$ Parry arcs, Figure 14.5 . These halos are rare, but they are more akin to the odd radius halos in the next chapter than are the more common halos that we could have considered instead, and this is why we have chosen them as illustrations. The figure shows their poles, together with a point of the unit sphere marked $\mathbf{D}_{u}$, the so-called minimum deviation (entry) point. The point $\mathbf{D}_{u}$ is on the meridian $(y=0)$ at an angle $\Delta_{\text {min }} / 2=23^{\circ}$ below the equator.

The figure also shows the halos themselves. Remarkably, the direction of each halo from the sun is nearly the same as the direction of its pole from the point $\mathbf{D}_{u}$. And since the wedge angle is $90^{\circ}$, the halos are all $46^{\circ}$ or a bit more from the sun. So from the pole diagram and the wedge angle you can see at once where to look in the sky for each halo.

The halos in Figure 14.5 - the $46^{\circ}$ Parry arcs-are examples of non-contact arcs. The close relation between the poles and the halos holds not just for the halos in the figure but for non-contact arcs in general. The location of the pole of a non-contact arc therefore gives a natural way to classify and describe the arc. Thus the arc is a left arc or right arc according to whether its pole is on the left or right hemisphere. The arc is symmetric if its pole is on the meridian $(y=0)$ of the sphere, because it turns out that such halos are indeed left-right symmetric. The arc is an upper arc or lower arc according to whether its pole is above or below the great circle passing through the points $\mathbf{D}_{u}$ and ( $0, \pm 1,0$ ) (Figure 14.6). Upper left and right arcs are also called supralateral arcs, and lower left and right arcs are called infralateral arcs.

We said that from the pole diagram you can tell where the halos appear in the sky. You can also anticipate the shapes of the halos, at least if the tilts of


FIGURE 14.5 (Top left) Poles (black dots) of $46^{\circ}$ Parry arcs. Each pole is a point on the unit sphere. The minimum deviation point $\mathbf{D}_{u}$ is located $\Delta_{\text {min }} / 2$ below the equator as shown. (Top right) Same but in stereographic projection looking down the $x$-axis. (Bottom) The $46^{\circ}$ Parry arcs themselves. The direction of each arc from the sun is about the same as the direction of its pole from $\mathbf{D}_{u}$. Depending on sun elevation $\Sigma$, an arc can be empty, that is, it does not appear. For $\Sigma=20^{\circ}$ (left) the lower symmetric arc (i.e., the circumhorizon arc) is empty, whereas for $\Sigma=70^{\circ}$ (right) the other five arcs are empty.


FIGURE 14.6 Terminology for non-contact arcs. Each non-contact arc is classified according to the location of its pole on the unit sphere. The dividing curve between right and left arcs is of course the meridian $y=0$, but the dividing curve between upper and lower arcs is the great circle passing through the points $\mathbf{D}_{u}$ and $(0, \pm 1,0)$ (not the equator $z=0$ ). The suncave arcs are those having poles close to $\mathbf{D}_{u}$, and the sunvex arcs are those having poles far from $\mathbf{D}_{u}$.
the crystals are small. If a non-contact arc has its pole fairly far from both the minimum deviation point $\mathbf{D}_{u}$ and its antipode $-\mathbf{D}_{u}$, then, when the arc is nonempty, it will be a more or less U-shaped curve convex toward the sun; such arcs are referred to as sunvex. On the other hand, if the arc has its pole close to $\mathbf{D}_{u}$ (or $-\mathbf{D}_{u}$ ), then, when it is non-empty, it will be concave toward the sun; such arcs are suncave. Thus the poles of sunvex arcs occupy a broad and vaguely defined band of the sphere about $90^{\circ}$ from $\mathbf{D}_{u}$ - this is the region far from $\mathbf{D}_{u}$ and $-\mathbf{D}_{u}$ while the poles of suncave arcs occupy the region close to $\mathbf{D}_{u}$ (Figure 14.6). An example of a sunvex arc would be the upper right $46^{\circ}$ Parry arc, and an example of a suncave arc would be the lower symmetric $46^{\circ}$ Parry arc, both shown in Figure 14.5.

There is more. Again we use the $46^{\circ}$ Parry arcs as examples. Each of these arcs, as we said, is on or just outside the $46^{\circ}$ halo. But for each there is just one sun elevation $\Sigma$ for which the arc actually contacts the $46^{\circ}$ halo. The zenith angle $\sigma$ of the sun at that moment is the same as the angular distance $s$ from $\mathbf{D}_{u}$ to the pole $\mathbf{P}_{u}$ of the arc; that is, $\sigma=s$, and therefore $\Sigma=90-s$. And the direction of the contact point from the sun is exactly the direction of $\mathbf{P}_{u}$ from $\mathbf{D}_{u}$. This is illustrated in Figure 14.7 for the upper right $46^{\circ}$ Parry arc.

A much easier example would be the upper symmetric $46^{\circ}$ Parry arc, whose pole we found in Figure 14.3. Can you find the sun elevation for contact and then the location of the contact point on the $46^{\circ}$ halo? Hint: Figure 14.5.

Because for any non-contact arc the angular distance $s$ from $\mathbf{D}_{u}$ to $\mathbf{P}_{u}$ gives the solar zenith angle for which the arc contacts the circular halo, it also gives an indication of the solar zenith angles for which the arc is non-empty, that is, for which it can appear. Roughly: the arc is non-empty for solar zenith angles close to $s$ and is empty for solar zenith angles far from $s$. Thus a suncave arc will appear when the sun is high overhead, since its pole is close to $\mathbf{D}_{u}$, whereas a


FIGURE 14.7 Upper right $46^{\circ}$ Parry arc, showing the close relation between its pole and its contact point with the $46^{\circ}$ halo. (Left) The pole $\mathbf{P}_{u^{\prime}}$ viewed looking directly down at the minimum deviation point $\mathbf{D}_{u}$ from outside the unit sphere. The angular distance $s$ of $\mathbf{P}_{u}$ from $\mathbf{D}_{u}$ is $79^{\circ}$. (Middle) Showing the arc for the one sun elevation $\Sigma=90-s=11^{\circ}$ for which it contacts the $46^{\circ}$ halo. The direction of the contact point $\mathbf{H}$ from the sun $\mathbf{S}$ is exactly the same as the direction of $\mathbf{P}_{u}$ from $\mathbf{D}_{u}$. This relation holds for all non-contact arcs. (Right) Same but for $\Sigma=22^{\circ} \neq 11^{\circ}$. The arc does not quite contact the circular halo.
sunvex arc will appear when the sun is low, since its pole is far from $\mathbf{D}_{u}$. But the wedge angle also plays a role, since halos arising from small wedge angles tend to be visible over a wider range of sun elevations than those arising from larger wedge angles.

From the preceding paragraph you can see that a non-contact arc with pole far to the rear of the sphere can only appear when the sun-or other light source-is below the horizon. Such a situation might arise when sunlight reflects from a frozen lake surface and then encounters ice crystals in the atmosphere; the light source is then the sun's reflected image, which is below the horizon. Such situations are obviously unusual, and in this book we ignore non-contact arcs whose poles are far to the rear of the sphere.

Much of what we said in this chapter depends on the tilts of the crystals being very small. In practice some qualification is needed, since small tilts are the exception rather than the rule. With even moderate tilts, non-contact arcs will contact the circular halo for a broad range of sun elevations, rather than at just one elevation. And for moderate to large tilts, the arcs tend to become circular, so that the distinction between sunvex and suncave is lost.

Reminder: In this section we have been talking about non-contact arcs. Contact arcs will be treated later.

