Text Supplement for ScholarWorks@UA collection Curves $\hat{V}_\gamma(\omega)$ (*Tape and Tape*, 2017) for download and details of their calculation

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Attribution: This collection of files is available online (*Tape*, 2017).
If you use these files, please cite *Tape and Tape* (2017).

Updates in Version 2

All files in this ScholarWorks collection were calculated using equations that appeared in the manuscript that was originally submitted to GJI for review (available on request). Although those equations are correct, they are not quite the same as the corresponding, more conceptual, equations in the paper that was eventually published. We performed a few numerical checks between the new and old equations to confirm that the results agreed with each other, but we did not recalculate the full set of $\hat{V}_\gamma(\omega)$ curves using the new equations.

S1 Description of files

The files in this collection are a supplement to *Tape and Tape* (2017). Here we provide pre-computed curves $\hat{V}_\gamma(\omega)$, where $\hat{V}_\gamma(\omega)$ is defined as in Section 2.4 of *Tape and Tape* (2017). The motivation for this work is the characterization of uncertainties for moment tensors for events in Earth's crust, such as earthquakes, volcanic events, and more. The curves $\hat{V}_\gamma(\omega)$ were calculated using Matlab's `integral2` function and have an accuracy of approximately 2×10^{-7}.

- `Vhat_scholarworks.pdf` [this file]
  Figures S1–S3 pertain to the time it takes to calculate $\hat{V}_\gamma(\omega)$ using Matlab's `integral2`. Figures S4–S6 pertain to the accuracy of $\hat{V}_\gamma(\omega)$, based on the difference between $\hat{V}_\gamma(\omega)$ and $\hat{V}_{-\gamma}(\omega)$.
- `Vhat_gammap_atol10.zip`
  zipped set of ASCII files of $\hat{V}_\gamma(\omega)$ and $\hat{V}_{-\gamma}(\omega)$ for $\gamma = 0°, 0.5°, \ldots, 29°, 29.5°, 30°$ and $\omega = 0°, 0.1°, \ldots, 179.9°, 180.0°$.
- `Vhat_gamman_atol10.zip`
  Same as previous, but for $\gamma = 0°, -0.5°, \ldots, -29°, -29.5°$.
- `omega_crit.zip` (see below)
- `Vhat_gammap_all.pdf` [example figure shown in Figure S1]
- `Vhat_gamman_all.pdf` [example figure shown in Figure S1]
- `Vhat_gamma_diff.pdf` [example figure shown in Figure S4]

S2 Calculation of $\hat{V}_\gamma(\omega)$

We calculate $\hat{V}_\gamma(\omega)$ curves using Matlab's two-dimensional integration function, `integral2`, in Matlab version 8.3.0.532 (R2014a). Matlab's `integral2` function has three optional arguments: the absolute tolerance, the relative tolerance, and the type of integration method, which is either `tiled`, `iterated`, or `automatic`. The default values for `integral2` are 10^{-10} , 10^{-6}, and `automatic`.

From the Matlab documentation: *The integral2 function attempts to satisfy:*

```
abs(q - Q) <= max(AbsTol,RelTol*abs(q))
```

where `q` *is the computed value of the integral and* `Q` *is the (unknown) exact value. The absolute and relative tolerances provide a way of trading off accuracy and computation time. Usually, the relative tolerance determines the accuracy of the integration. However if* `abs(q)` *is sufficiently small, the absolute tolerance determines the accuracy of the integration. You should generally specify both absolute and relative tolerances together.* We used the default absolute tolerance of 10^{-10}, a relative tolerance of 10^{-18}, and the `iterated` integration method. With these setting, the resulting $\hat{V}_\gamma(\omega)$ curves were accurate to at least 10^{-7} (see next section).

S3 Accuracy of $\hat{V}_\gamma(\omega)$

We have two methods for verifying that the $\hat{V}_\gamma(\omega)$ curves are accurate. The first method is rough, the second is more precise.

1. Generate a large ($N \approx 10^7$) set of uniform moment tensors M with fixed source type $\mathbf{\Lambda} = \mathbf{\Lambda}(\delta, \gamma)$, $\delta = 0°$. Choose any moment tensor in the set as the reference moment tensor M_0, calculate $\omega = \angle(M_0, M)$, and plot a normalized histogram of ω-values. The histogram should approach $\hat{V}'_\gamma(\omega)$ for $N \to \infty$ and bin widths $\to 0$.

2. For $\gamma > 0$ calculate $\hat{V}_\gamma(\omega)$ and $\hat{V}_{-\gamma}(\omega)$. From theory (*Tape and Tape*, 2016),

$$\Delta\hat{V}_\gamma(\omega) = \hat{V}_\gamma(\omega) - \hat{V}_{-\gamma}(\omega) = 0 \tag{S1}$$

   Since $\hat{V}_\gamma(\omega)$ and $\hat{V}_{-\gamma}(\omega)$ are calculated differently, we can use the calculated difference as an approximation for the accuracy of either one.

Calculation of $\hat{V}_{-\gamma}(\omega)$ is considerably slower than calculation of $\hat{V}_\gamma(\omega)$, due to the more extreme integration surfaces (*Tape and Tape*, 2017).

Using the two sets of curves, $\hat{V}_{-\gamma}(\omega)$ and $\hat{V}_\gamma(\omega)$, we calculate $\Delta\hat{V}_\gamma(\omega)$ for $\gamma = 0.5°, 1°, 1.5°, \ldots, 29°, 29.5°$. Plots related to $\Delta\hat{V}_\gamma(\omega)$ are shown in Figures S4–S6. There are clearly some systematic patterns in $\Delta\hat{V}_\gamma(\omega)$ (Figure S5) that are probably related to the level surfaces of either $\hat{V}_\gamma(\omega)$ or $\hat{V}_{-\gamma}(\omega)$, but we did not attempt to understand the patterns.

S4 Plotting $\hat{V}_{\mathbf{\Lambda}}(\omega)$ curves for any moment tensor source type $\mathbf{\Lambda}$

The precomputed $\hat{V}_\gamma(\omega)$ are for deviatoric source types only. For an arbitrary source type $\mathbf{\Lambda}$, the function $\hat{V}_{\mathbf{\Lambda}}(\omega)$ can be reduced to the deviatoric case using Eq. (45) of *Tape and Tape* (2016). Some interpolation is required, since the precomputed $\hat{V}_\gamma(\omega)$ are only available for discrete γ and ω.

The main steps to use the precomputed curves are:

1. Download the repository `compearth` from github, following the instructions here:

   `https://github.com/carltape/compearth`

2. Unzip the three sets of files (`omega_crit.zip`, `Vhat_gamman.zip`, `Vhat_gammap.zip`) in some 'base' directory for permanent archiving.

3. Open Matlab from inside `compearth` in the directory `compearth/momenttensor/matlab/`

4. Specify your base directory path in `Vhat/path_Vhat.m`

5. Run `load_mt.m` to set all paths.

6. Open the file `Vomega.m` and try some of the examples at the bottom; these examples call the function `Vomega.m` via the function `plot_Vomega.m`, which has several plotting options inside.

   Note that all precomputed $\hat{V}_\gamma(\omega)$ curves can be loaded with `load_Vgammaomega.m`

S4.1 $\hat{V}'(\omega)$ at critical values

The file `omegacrit.zip` contains ASCII files such as `omegacrit_100gamma_2750.dat`:

```
  0.000000000000     0.000000000000     0.000000000000
  5.000000000000     0.029122778948     0.000636133855
115.000000000000     1.727997921308     0.784101016778
120.000000000000     1.779267176261     0.936843935829
125.000000000000     0.000000000000     1.000000000000
```

The first column gives the critical values $\omega_0, \ldots, \omega_4$ (in degrees), the second column is $\hat{V}'_\gamma(\omega)$, and the third column is $\hat{V}_\gamma(\omega)$. The example above is for $\gamma = 27.5°$. The values of $\hat{V}'_\gamma(\omega)$ were calculated using the numerical derivative formula

$$\hat{V}'_\gamma(\omega) \approx \frac{\hat{V}_\gamma(\omega + \Delta\omega/2) - \hat{V}_\gamma(\omega - \Delta\omega/2)}{\Delta\omega}$$

with $\Delta\omega = 10^{-6}$ radians and with the following settings for Matlab's `integral2`: absolute tolerance 10^{-12}, relative tolerance 10^{-18}, `iterated` integration.

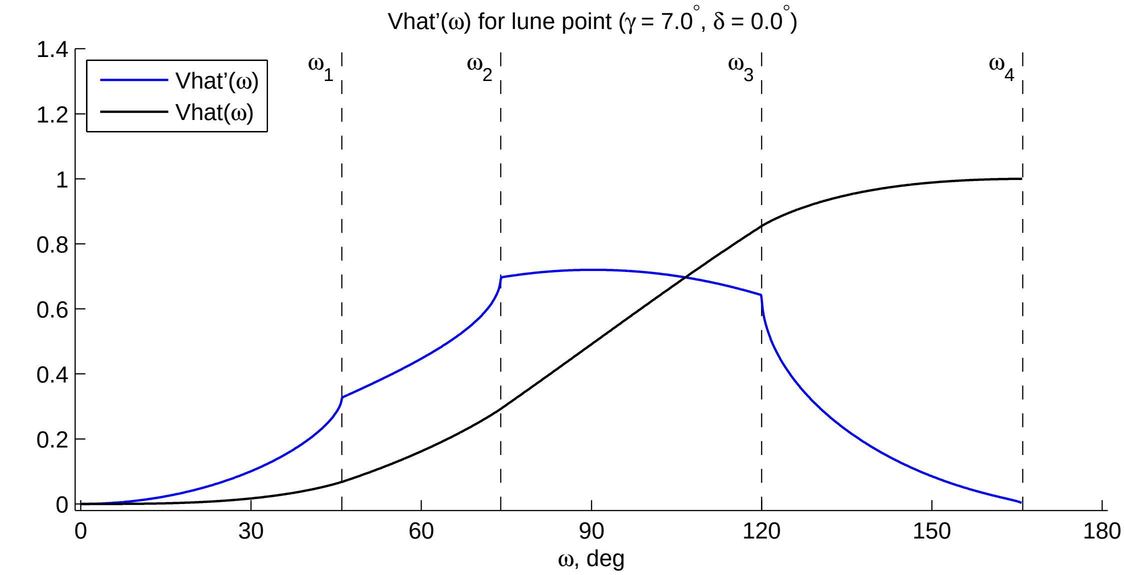
References

Tape, C. (2017), Curves $\hat{V}_\gamma(\omega)$ (Tape and Tape, 2017) for download and details of their calculation, ScholarWorks@UA at `http://hdl.handle.net/11122/7234` (last accessed 2017-01-30): descriptor file, figures, and curves in ASCII format.

Tape, W., and C. Tape (2015), A uniform parameterization of moment tensors, *Geophys. J. Int.*, *202*, 2074–2081, doi:10.1093/gji/ggv262.

Tape, W., and C. Tape (2016), A confidence parameter for seismic moment tensors, *Geophys. J. Int.*, *205*, 938–953, doi:10.1093/gji/ggw057.

Tape, W., and C. Tape (2017), Volume in moment tensor space in terms of distance, *Geophys. J. Int.*, *210*, 406–419, doi:10.1093/gji/ggx164.



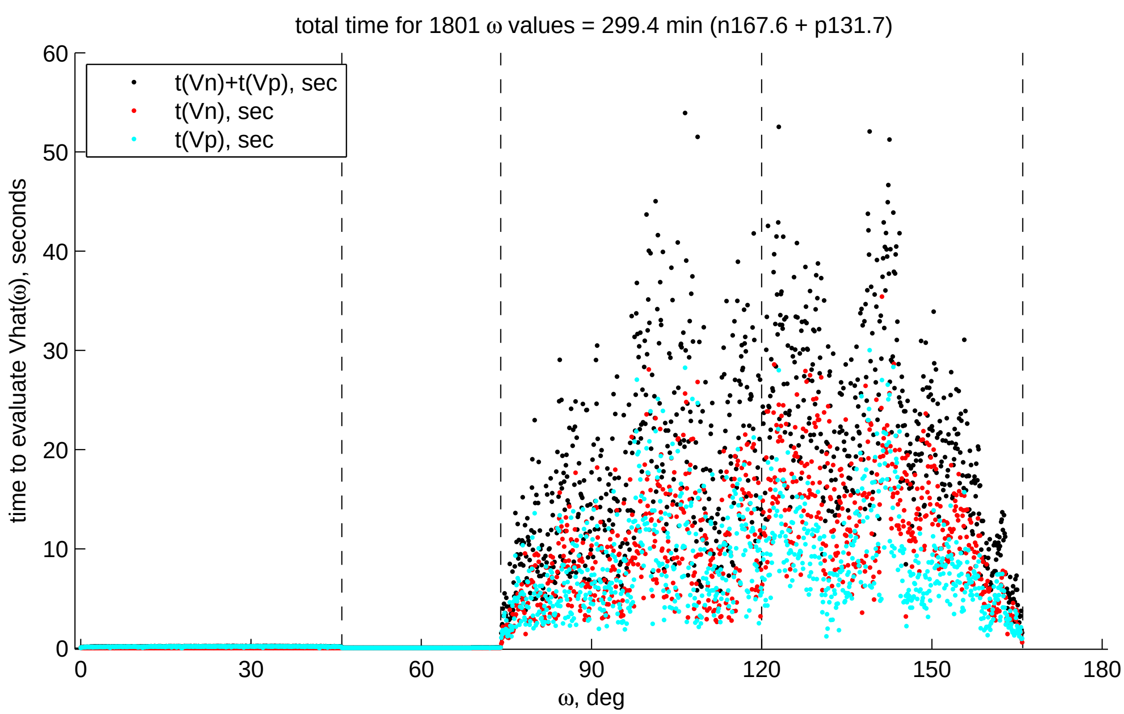


Figure S1: Example calculation of $\hat{V}_\gamma(\omega)$ for $\gamma = 7°$. A set of these figures is included within the ScholarWorks collection as `Vhat_gammap_all.pdf`. A corresponding set of these figures, for $\gamma < 0$, is included as `Vhat_gamman_all.pdf`. (top) $\hat{V}_\gamma(\omega)$ (black) and $\hat{V}'_\gamma(\omega)$ (blue), with the critical values $\omega_1(\gamma), \ldots, \omega_4(\gamma)$ marked as vertical dashed lines. (bottom) Time to evaluate $\hat{V}_\gamma(\omega)$, in seconds. Each evaluation is achieved by summing two different integrations. In this example, the total time to evaluate 1801 values is 299.4 minutes. The integration time depends strongly on the integration tolerances that are chosen.

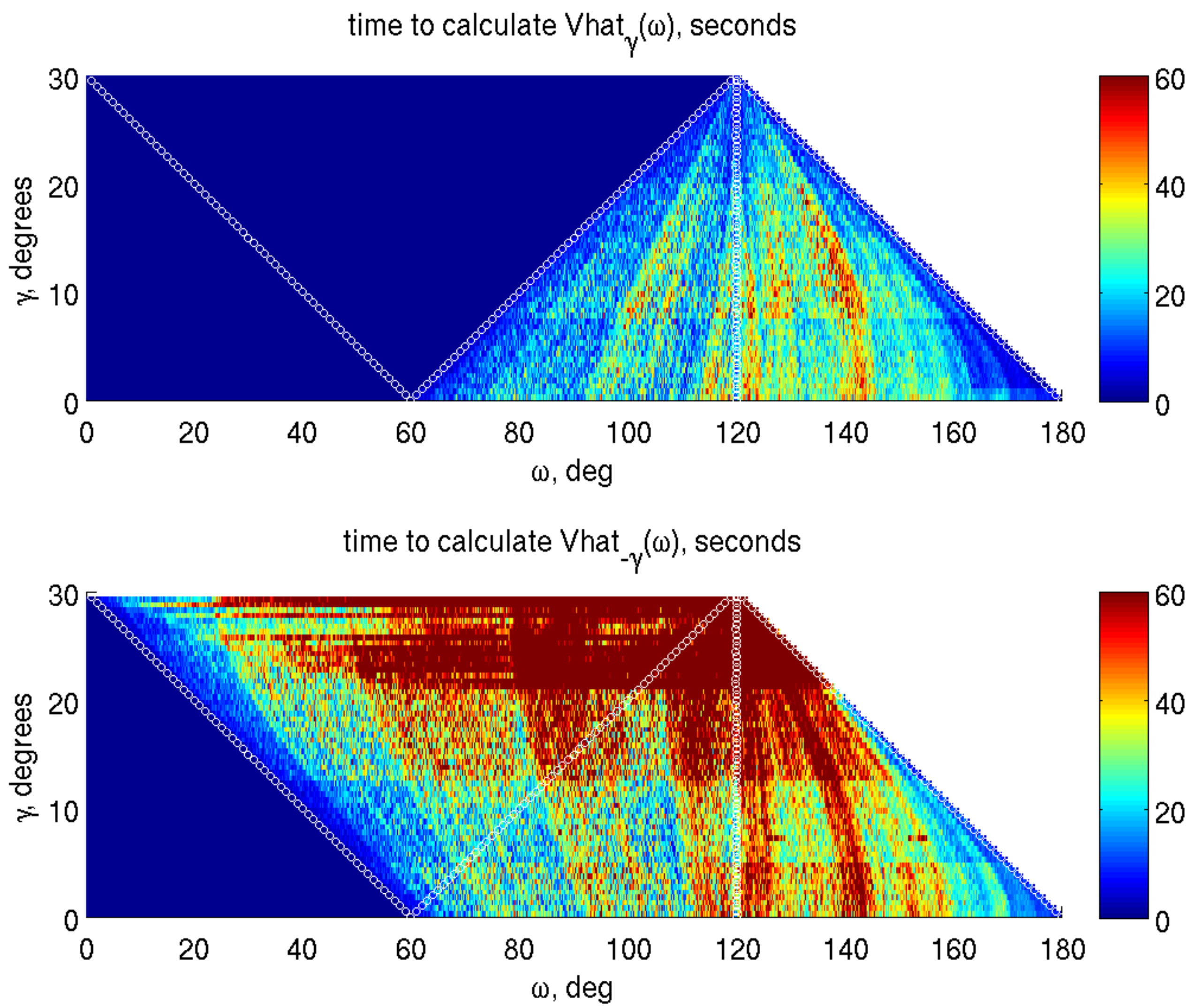


Figure S2: Time to calculate $\hat{V}_\gamma(\omega)$ (top) and $\hat{V}_{-\gamma}(\omega)$ (bottom), in seconds, for $\gamma > 0$. The white circles mark the critical values, from left to right, ω_1, ω_2, $\omega_3 = 120°$, and ω_4. (See Figures 7 and 8 of *Tape and Tape* (2015) for insights on critical values.) The horizontal streaking arises because different computers were used to calculate different γ values. Clearly the $\hat{V}_{-\gamma}(\omega)$ points take longer to calculate than the $\hat{V}_\gamma(\omega)$ points.

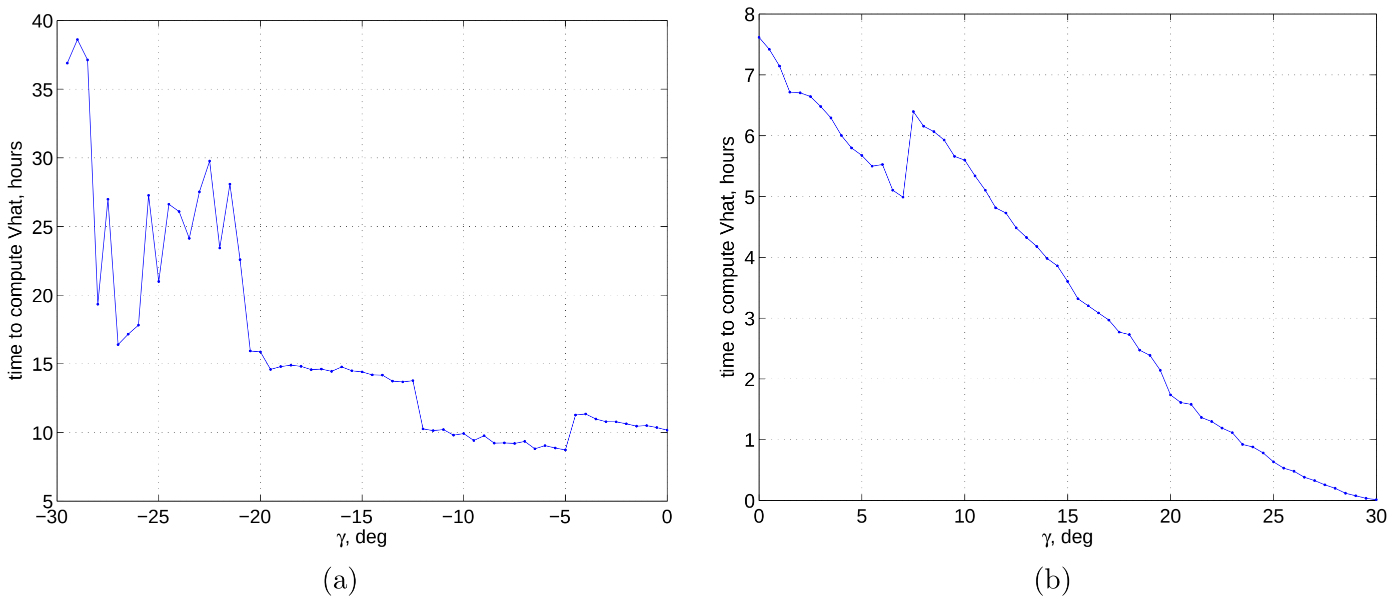
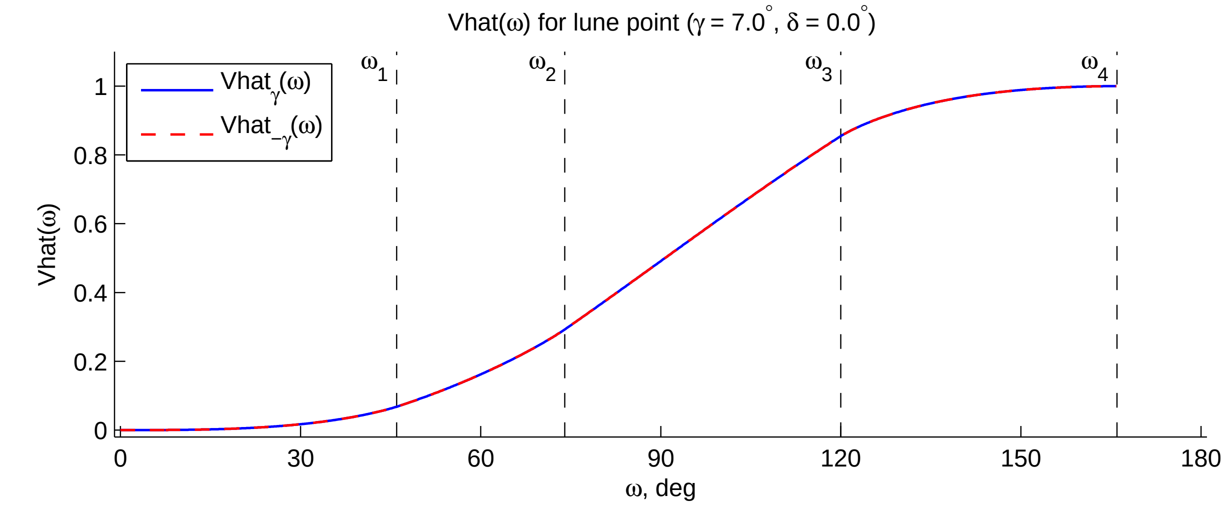


Figure S3: Time to calculate $\hat{V}_\gamma(\omega)$ curves, in hours, for $\gamma < 0$ (a) and $\gamma \geq 0$ (b). Note the difference in the y-axis scale for (a) and (b): it appears always to be faster to compute $\hat{V}_\gamma(\omega)$ than $\hat{V}_{-\gamma}(\omega)$.



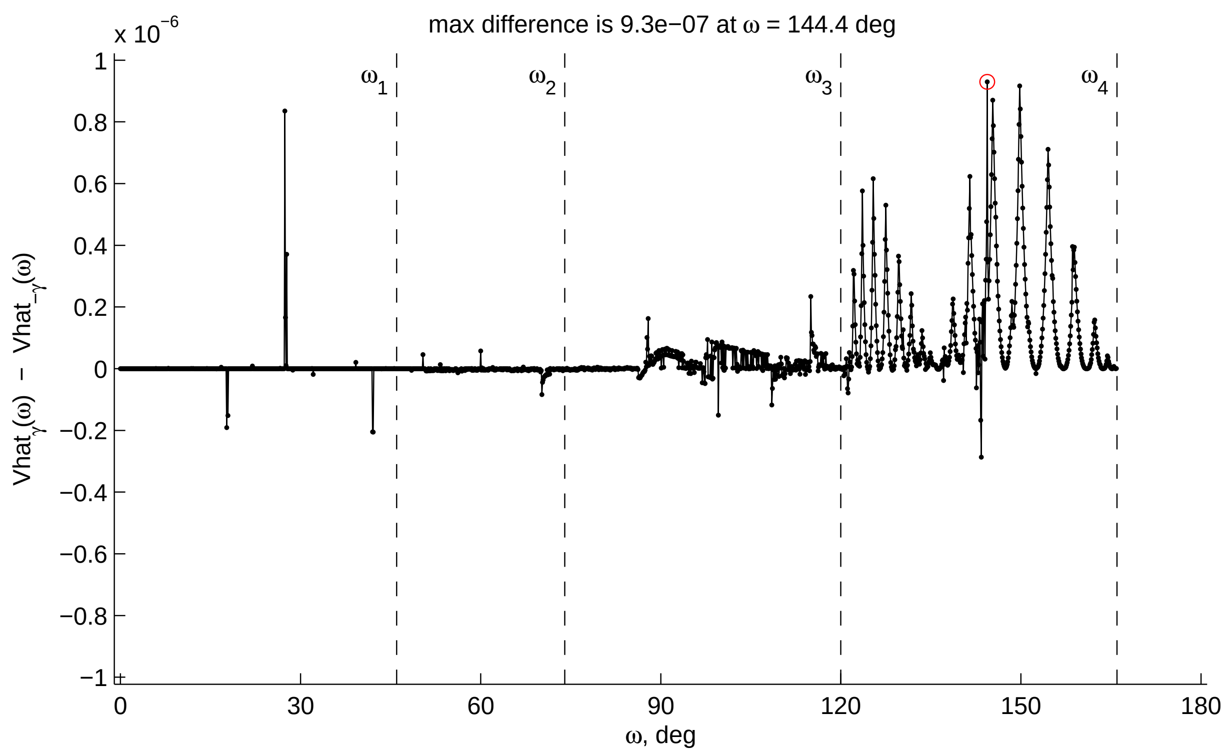
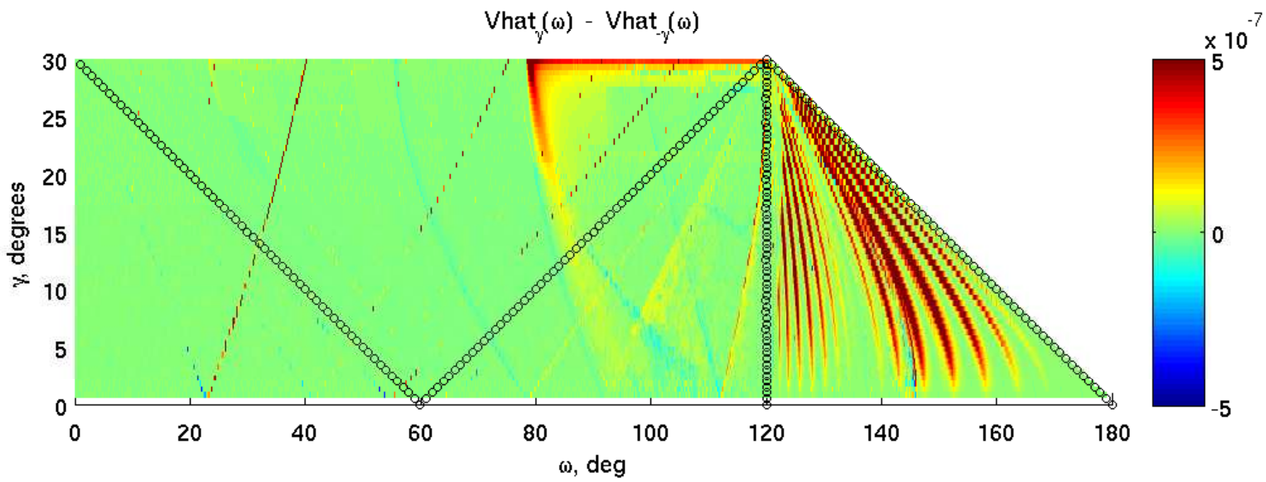
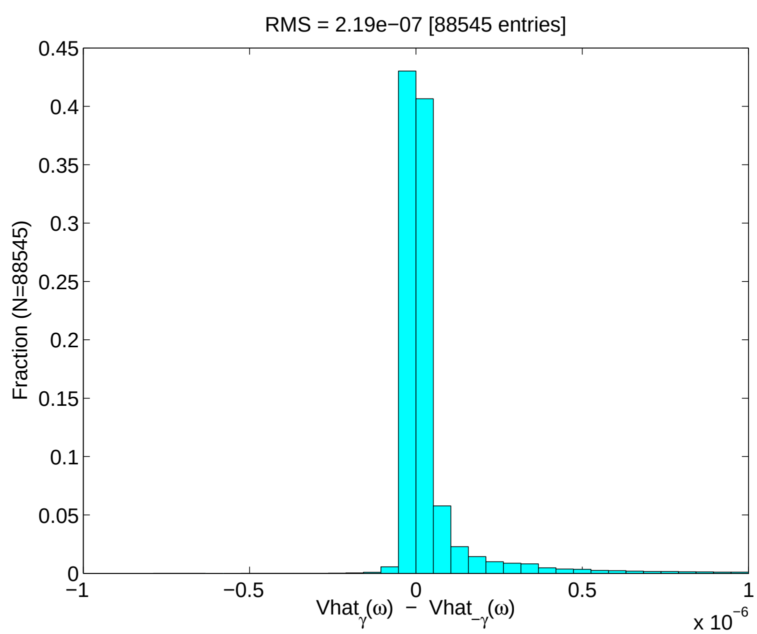


Figure S4: $\hat{V}_\gamma(\omega)$ (blue) and $\hat{V}_{-\gamma}(\omega)$ (dashed red) for $\gamma = 7°$. In theory, these two curves are identical, but there are small differences due to the limited accuracy of the numerical integration. The lower diagram shows the difference $\hat{V}_\gamma(\omega) - \hat{V}_{-\gamma}(\omega)$, which has a maximum of 9.3×10^{-7} (red circle). Vertical dashed lines are at the critical values $\omega_1, \ldots, \omega_4$. A set of these figures is included within the ScholarWorks collection as `Vhat_gamma_diff.pdf`.

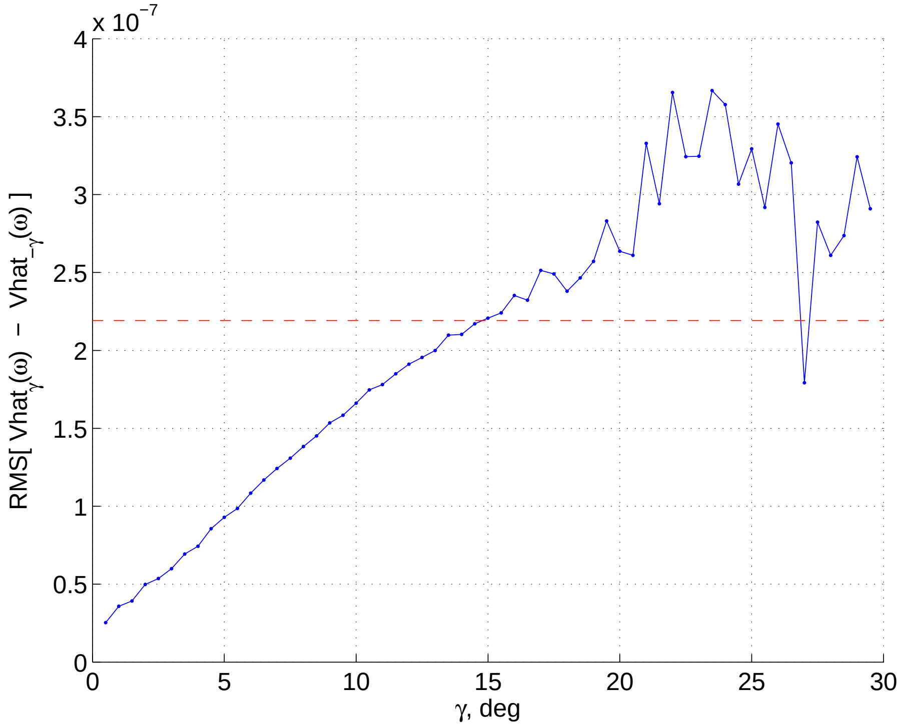


(a)

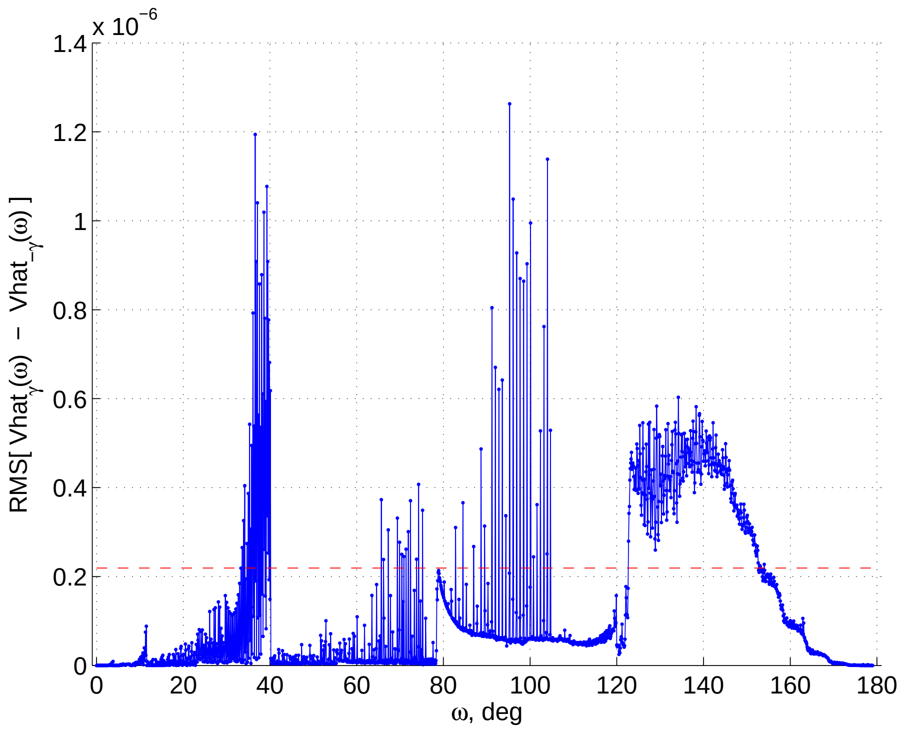


(b)

Figure S5: (a) $\hat{V}_\gamma(\omega) - \hat{V}_{-\gamma}(\omega)$ for all $\omega = 0.0°, 0.1°, \ldots, \omega_4(\gamma)$ and for all $\gamma = 0.5°, 1.0°, \ldots, 29.5°$. The black circles mark the critical values, from left to right, ω_1, ω_2, $\omega_3 = 120°$, and ω_4. (b) Histogram of all differences, with a root-mean-squared of 2.2×10^{-7}.



(a)



(b)

Figure S6: Root-mean-squared (RMS) of values of $\Delta\hat{V}_\gamma(\omega) = \hat{V}_\gamma(\omega) - \hat{V}_{-\gamma}(\omega)$ for fixed γ (top) and for fixed ω (bottom). The red dashed line is the overall RMS value of 2.2×10^{-7} (Figure S5). (a) RMS as a function of γ. For each γ we calculate the RMS of $\Delta\hat{V}_\gamma(\omega)$ for all ω values. (b) RMS as a function of ω. For each ω we calculate the RMS of $\Delta\hat{V}_\gamma(\omega)$ for all γ values.