Introduction

Chatic behavior in a spatially extended system is often referred to as spatio-temporal chaos. The trajectories of a system as it evolves through state space are characterized by irregular spatial and temporal patterns.

In mathematical biology, spatio-temporal chaos has been demonstrated in insect models (Parlitz & Wittenberg, 1992) and in models of the Hodgkin-Huxley nerve cell model (Smith, J. & Fowles, A., 1995) and the Hodgkin-Huxley neural model (Wang, L. & Chen, X., 2003).

Transient chaos is a special case of chaotic dynamics in which the system dynamics collapses without external perturbation. Rather, collapse is an intrinsic property of the system. Here, we describe how spiking neurons in the brain can synchronize and the network can collapse on to two different species of attractor: the limit cycle and the steady-state solution.

The Morris-Lecar Model

The Morris-Lecar model was derived empirically from the barnacle giant muscle fiber (Morris & Lecar, 1981). The system equations describe the membrane potential (V) of a neuron as a function of ion currents and the state of the potassium channel (n). The applied current (Iapp) is the bifurcation parameter.

\[ CV = I_{app} - I_c - I_k - I_l \]
\[ n = n_{ss} \]
\[ I_c = g_C(V - V_Ca) \]
\[ I_k = g_K(V - V_K) \]
\[ M = \frac{1}{1 + \tanh \left( \frac{V - V_Ca}{1} \right)} \]
\[ n_s = \frac{1}{1 + \tanh \left( \frac{V - V_K}{1} \right)} \]
\[ r = \frac{1}{\tau_C(V - V_Ca)} \]

The channel equations describe each current's value across the membrane at any time, its time course of potassium channel activation.

\[ V_c = V_c + I_{app} - I_c - I_k - I_l \]
\[ n = n_{ss} \]

In the parameter range of interest, the Morris-Lecar system has three fixed points. A stable (blue) and an unstable (red) fixed point. The unstable node above the bifurcation parameter represents a saddle-node bifurcation, which results in the intersection points colliding and annihilating each other as Iapp approaches 38.8. The saddle-node bifurcation is a special case of chaotic dynamics in which the system dynamics collapses without external perturbation. Rather, collapse is an intrinsic property of the system.

Stability Analysis

The nullclines of the system describe each of the system equations. They are normal to each other and they cross at a fixed point. The nullclines of the system describe where each of the system equations intersect. The nullclines are the bifurcation parameter, which represents a saddle-node bifurcation.

Electrically Coupled Neural Network

A coupled ring network consists of several Morris-Lecar neurons. Each neuron's membrane potential is coupled to the membrane potential of its neighbor through a diisulfide coupling term.

\[ V_i = \frac{1}{N} \sum_{j=1}^{N} V_j + D(V_i - V_{j-1} - 2V_i) \]

The coupling term allows information to spread across the network through neighboring interactions. In a network at resting potential, neurons moved to the excitable region are called kicker neurons.

Figure: A single-kicker neuron spreads activity through the network (right). The vertical axis represents the time course of spikes in each neuron, and the horizontal axis represents the time course of potassium channel activation. Below: the effect of network size on the bifurcation parameter is increased. Starting from the left, Iapp=28.5, 30, 32, 38.

Transient Basins of Attraction

A basin of attraction gives the set of initial conditions for which a particular attractor is reached. Here, the 'transient' basin of attraction is computed. A set of neurons is moved to a point in phase space and the network is allowed to evolve deterministically for 1000m. At that point, the state of the network is determined. Only a particular subset of neurons are represented by the color of the phase point and the state of other neurons in the network are either at resting potential, chaotic state or are located on the chaotic saddle, depending on the study (see Basins Figures 2 & 3).

Mechanisms of Collapse

A transient basin study demonstrates two distinct mechanisms for collapse in the diffusively coupled ring network. For a network composed of neural networks, these mechanisms are more probable for higher current values, but for an active network released on the attractor, the lower current values carry a higher probability of collapsing into the limit cycle. As the bifurcation parameter is increased, the probability of periodic collapse begins to spread to higher applied currents. On the other hand, each large synchronized perturbation is unlikely in the closed system that experiences no external perturbation with the exception of a network of 28 neurons. In these cases, most networks fall to the rest state except a small cluster of neurons that may either remain active. This network, or collapse its dynamics. This network collapse is associated with a higher chance of collapse to the periodic state. These data suggest distinct collapse mechanisms depend on the parameter value.

Discussion

The empirically derived Morris-Lecar neuron model makes a powerful element for a network study, allowing a compromise between complexity and computational efficiency. The system can express drastic changes over short temporal scales, shifting from an active chaotic state to simple periodic or steady-state behavior. Correlation studies reveal no long-term spatial or temporal correlation between members of the ensemble and collapse comes suddenly, without warning. System collapse is an intrinsic property, occurring in the absence of external influence. Chaos can persist in the network for long lifetimes, but the system always inevitably collapses to either a periodic (red) or steady (black) state. That complicated interaction with a binary result can occur in a diffusively-coupled system of spiking neurons is not a significant phenomenon for gap junctions in neural computations, especially with respect to lateral information transfer in parallel wired transmission. Gap junctions serve as gates between cells. In the CNS, both astrocytes and neurons rely on gap junctions for intercellular communication. In neurons, gap junction coupling is sufficient to evoke action potentials that can synchronize activity across a network (Vakayama & Carlsson, 2000). Gap junctions have been found on perisynaptic and perisomatic locations in the mouse hippocampus (Haeusser, Schoppa, et al., 2007) and throughout the adult cerebral cortex (Braga, et al., 2003). They have also been shown, in conjunction with inhibitory synapses, to modulate inhibitory neurotransmitter (NMDA, GABA) localization (Roth-Carlin, B. & Berger, A., 2000).