Highly incident configurations and reduced Levi graphs

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### What is a configuration?

An \((n,k)\)-configuration is a collection of points and lines in the Euclidean plane where each point lies on \(n\) lines and each line passes through \(k\) points.

If \(n=k\), we call it a \(k\)-configuration.

Figure 1 is a 3-configuration; each line passes through 3 points and each point lies on 3 lines. The configuration in Figure 1 has 3-fold rotational symmetry which allows us to construct its reduced Levi graph.

### Methods

To find a reduced Levi graph of a larger configuration, we did the following to each graph in Figure 3.

1. Label one of each double edge with a 0.
2a. Label a 6-cycle with \((a,b,c,a,b,c)\).
2b. Label an 8-cycle with either \((a,b,c,d,a,d,c)\) or \((a,d,c,a,b,c,d)\).
3. Force all other 6-cycles to be trivial, that is, of the form \((a,b,c,a,b,c)\) or all other 8-cycles to be either \((a,b,c,d,a,d,c)\) or \((a,d,c,a,b,c,d)\).
4. Check graphs for equal adjacent edges.

### Results

This investigation did not result in any labelings that correspond to a reduced Levi graph. Here are a few of the results for each graph. Note the doubled edges are not shown for clarity.

#### Nauru Graph

The Nauru graph with doubled edges produces a labeling with no equal adjacent edges. However, the 8-cycles are of the form \((a,d,f,c,a,y,e,x)\). This 8-cycle does not correspond to a trivial configuration.

#### (4,8)-configurations

During this investigation, we found three previously undiscovered \((4,8)\)-configurations. Figure 7 gives one of them and its reduced Levi graph.

### Open Questions

1. Are there choices of labels for the Nauru graph that result in the reduced Levi graph of a configuration?
2. What other graphs with each edge doubled might be a reduced Levi graph for a configuration?

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