Odd radius plate arcs arise in pyramidal crystals having plate orientations. These arcs are rare, but not so rare as you may imagine. They are seen much more often than odd radius circular halos, for example.

Mostly we will restrict the discussion to refraction arcs, that is, to arcs involving refraction only; these halos are by far the most important. And as in Chapter 8, where we studied odd radius circular halos, the crystals that we will use to explain the odd radius plate arcs are pyramidal crystals with \{10\bar{1}1\} faces (Figure 15.1). As explained in Chapter 12, the plate arcs resulting from these crystals fall into eight families—the 9° plate arcs, the 18° plate arcs, the 20° plate arcs, the 22° plate arcs, and so forth—one family for each of the circular halos of Table 8.1. Here we are including the familiar 22° and 46° plate arcs; although they are not, strictly speaking, odd radius, they do sometimes form in pyramidal crystals.

First let’s look at some photographs of odd radius plate arcs. Later we will explain in more detail why we think we know what we are seeing in such photographs.

At first glance the halo display in Figure 15.2 appears quite ordinary—just a pair of bright but ordinary parhelia. But had you been present at the time of the display, and had you used the extended-arm-and-spread-fingers method of Chapter 1 to estimate the angular distance of these arcs from...
FIGURE 15.2  The 18° plate arcs. Fairbanks, Alaska, March 3, 1999. $\Sigma = 14^\circ$.

FIGURE 15.3  Same display as Figure 15.2 but photographed thirteen minutes earlier. The two inner arcs are 18° plate arcs, and the outer arc is probably a halo intermediate between a 23° plate arc and a 23° circular halo. The colored dots are at $\Delta_{\min} = 18.3^\circ$, 21.8°, and 22.9° from the sun (Table 8.1).
the sun, you would have found that they were too close to the sun to be parhelia. And had you been present ten minutes earlier, you would have seen the same pair of bright arcs but accompanied by a nearly circular arc about five degrees further from the sun (Figure 15.3), making it obvious that some, at least, of the halos in this second figure must be odd radius. In fact, the two inner arcs are 18° plate arcs, and the outer arc is probably something intermediate between a 23° plate arc and a 23° circular halo.

Although that outer halo in Figure 15.3 may look like a 22° halo, it is not: Using the methods of Appendix D, we have positioned colored dots at the indicated angular distances from the sun. The green dot, 21.8° from the sun (\(\Delta_{\text{min}}\) for the 22° halo), is distinctly sunward of the halo in question; the halo cannot be the 22° halo. However, both in this figure and in the previous, pieces of what is probably the 22° halo do appear in the low crystal swarm just above the horizon.

Figure 15.4 shows a lunar display in which a number of bright stars happen to be located near the halos. Since the time and place of the photograph are known, the angular distances from the various stars to the moon can be calculated.

**Figure 15.4** Lunar halo display with 9°, 18°, and 23° plate arcs. The angular distance from the moon to each circled star is given. Fairbanks, December 5, 2003, 10:40 p.m. Moon elevation 41°.
accurately. From these angular distances it is evident that all of the halos in the photograph are odd radius. Even the bright arc at the top, which resembles the ordinary upper tangent arc (i.e., the 22° column arc) is an odd radius arc; the upper tangent arc would be \( \Delta_{\min} = 21.8^\circ \) above the moon, but the arc in the photo is nearly 23° above the moon—it is the upper 23° plate arc.

The 18° plate arcs are also present, to the left and right of the moon, and the lower 9° plate arc is present below the moon. Here these arcs appear as enhancements of their corresponding circular halos rather than as well-defined curves in their own right. This is typical; only in the best displays do odd radius plate arcs approximate the distinctive curves that would be expected from precisely horizontal crystals.

Figure 15.5 is a daytime display just a bit more complex than Figure 15.4. You can see immediately that at least some of the halos must be odd radius. The

**FIGURE 15.5** Halo display with odd radius plate arcs. Some circular halos are present as well, including the 20° halo, just outside the 18° plate arc. Vaala, Finland, May 12, 2002. \( \Sigma = 29^\circ \).
enhancements at various positions on the circular halos are signs that there is more here than just circular halos. The enhancements are plate arcs, namely, the lower 9°, the right 18°, the upper 23°, and the lower right 24° plate arcs.

Organizing the plate arcs

Figure 15.6 shows the theoretically expected plate arcs for sun elevations $\Sigma = 20^\circ$ and $\Sigma = 70^\circ$. Nearly all of the halos in the simulations are refraction arcs, and since they are plate arcs, they are non-contact arcs. They can therefore be understood in the context of the halo pole theory of the preceding chapter. Figure 15.7 is the relevant pole diagram; it shows the pole for each plate arc. From the pole and $\Delta_{\text{min}}$ we know what to expect for the corresponding arc, and it is therefore easy to correlate the poles in the pole diagram with the arcs in the simulation: The direction of the pole from the point $D_u$ gives the approximate direction of the arc from the sun, and $\Delta_{\text{min}}$ gives the approximate angular distance of the arc from the sun. If the pole is far from $D_u$, then the arc is sunvex and will appear when the sun is low. If the pole is close to $D_u$, then the arc is suncave and will appear when the sun is high. The lower 23° arc (wedge 13 2), for example, having its pole very close to $D_u$, appears only in the $\Sigma = 70^\circ$ simulation and is highly suncave. The upper 9° arc (wedge 13 6) and the two upper 24° arcs (13 5 and 13 7), having their poles fairly far from $D_u$, appear only in the $\Sigma = 20^\circ$ simulation and are sunvex. The upper 20° arc and the two upper 35° arcs, with poles even farther from $D_u$, appear in neither simulation; they would show up at very low sun elevation.

Simulations and halo poles are complementary approaches to understanding halos. Each approach does some things that the other cannot do. One of the things that the halo pole theory does is to exhibit all of the possibilities at once, in a single diagram. In the present case, all plate arcs are given in the pole diagram of Figure 15.7.\textsuperscript{1} This is something that no single simulation by itself can do. On the other hand, although the pole and the $\Delta_{\text{min}}$-value of a non-contact arc do in principle determine the arc, it is a highly idealized arc that they determine. Simulations give much more realistic results, as we will see momentarily.

Simulations of some real displays

Figure 15.8 is a first attempt at simulating the halo display of Figure 15.2. The crystals used in making the simulation were plate-oriented pyramidal crystals shaped like the one shown. They were given very small tilts, which resulted

\textsuperscript{1} Recall that we are talking about refraction arcs, we are assuming that the pyramidal crystal faces are the \{10\overline{1}1\} faces, and we are ignoring arcs with poles on the rear hemisphere.
FIGURE 15.6  Odd radius plate arcs. These simulations were made using plate-oriented pyramidal crystals shaped like the one in Figure 15.1. The tilts of the crystals were about 1°. Each refraction arc is labeled with its $\Delta_{\min}$ value, that is, with the radius of the corresponding circular halo. The true angular distance of an arc from the sun may be somewhat more than $\Delta_{\min}$, as in the left-hand simulation, where the 23° arc is actually about 28° from the sun. The tick marks, in a line emanating from the sun, are at one-degree intervals. (Left) $\Sigma = 20°$. (Right) $\Sigma = 70°$.

FIGURE 15.7  Poles of plate arcs. The responsible crystals have prism, basal, and {1011} pyramid faces as usual. The pole and $\Delta_{\min}$ determine the arc, as explained in the text; compare this figure with Figure 15.6. The minimum deviation point $D_u$ varies with $\Delta_{\min}$, but it is confined to the small elongated region marked “$D_u$.” (Left) Each pole is labeled with its $\Delta_{\min}$-value. (Right) Each pole is labeled with its wedge.
### FIGURE 15.8
A first attempt to simulate the halo display of Figure 15.2. The crystals used in making the simulation were shaped like the one shown and had plate orientations with very small tilts. The diagram at upper right depicts the crystal orientations, as explained in Figure 5.3.

<table>
<thead>
<tr>
<th>Ray path</th>
<th>Halo points</th>
<th>Halo</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>1.8%</td>
<td>Odd radius helic arc</td>
</tr>
<tr>
<td>23</td>
<td>6.3</td>
<td>Odd radius helic arc</td>
</tr>
<tr>
<td>1 23</td>
<td>33.6</td>
<td>Upper 23° plate arc</td>
</tr>
<tr>
<td>13 25</td>
<td>24.4</td>
<td>Left 18° plate arc</td>
</tr>
<tr>
<td>13 27</td>
<td>27.4</td>
<td>Right 18° plate arc</td>
</tr>
</tbody>
</table>

93.5%

### FIGURE 15.9
Same as Figure 15.8 but with larger crystal tilts. Large crystal tilts seem to be the norm for odd radius displays; odd radius arcs as well defined as those in Figure 15.8 are rare.
in unusually well-defined halos. Using the pole diagram of Figure 15.7, we can identify the halos in the simulation; they must be the left and right 18° plate arcs and the upper 23° plate arc. The direction of each arc from the sun is the same as the direction of its pole from $D_u$, as expected. The angular distance of the 18° arcs from the sun is close to 18°, also as expected. And although the angular distance of the 23° arc from the sun is considerably more than 23°, this could have been anticipated, since the sun elevation for contact with the 23° circular halo turns out to be 48°—substantially different from the sun elevation of the display.

Why don’t the remaining arcs with poles in Figure 15.7 appear in the simulation in Figure 15.8? Since there are no prism faces on the crystal, there are no 9°, 22°, 24°, or 46° arcs. The upper 20° plate arc is absent partly because its pole is far from $D_u$, so that the arc requires very low sun, and partly because the relatively large basal face 1 of the crystal interferes with the necessary ray path 13 16. The upper 35° plate arcs are absent because their poles are too far from $D_u$, and the lower 20° and 35° plate arcs are absent because they are below the horizon at the sun elevation of this display. (In low level displays in Fairbanks we do sometimes see the lower 35° plate arcs below the horizon, showing up as concentrations of sparkles between ourselves and the ground, sometimes in the space of only a few meters.) The lower 23° plate arc has pole close to $D_u$ and therefore occurs only at high sun.

The sorting of ray paths shown in the figure confirms that the halos in the simulation are mainly the 18° plate arcs and the upper 23° plate arc. Also present is a faint odd radius helic arc, formed by reflections from pyramid faces.

In Figure 15.9 the tilts of the crystals have been increased. The 18° plate arcs are no longer dramatically sunvex and instead look more like short pieces of the 18° circular halo. The 23° plate arc has become so diffuse and spread out as to nearly lose its identity. There is also a hint here of the upper 20° plate arc, and this is confirmed by a sorting of ray paths (not shown), which finds a few 13 16 ray paths—these paths are more realizable now, due to the larger tilts of the crystals.

Comparison of the simulation with the photograph of the display (Figure 15.2) makes us think we understand the display fairly well: We are seeing mainly the 18° plate arcs together with some diffuse and weak light that is the upper 23° plate arc.

The display in Figure 15.10 is more complex. The 18° and 23° circular halos are present, and the 20° circular halo is probably there as well, blending with the 18° halo. But there are plate arcs also, some of them quite subtle. The 18° and (probably) 22° plate arcs are bright and well defined, but the 24° plate arcs, near the purple dots, are fainter and more diffuse, and the 20° plate arc is fainter still. The 9° plate arc is not clearly distinguishable from the sun pillar. All of the plate arcs are where Figure 15.7 predicts.
FIGURE 15.10 Halo display with odd radius plate arcs. The colored dots are at angular distances of 18.3°, 21.8°, and 23.8° from the sun (Table 8.1). In the simulation, crystals like the upper one made the 9°, 18°, 20°, and 24° plate arcs, crystals like the middle one made the 22° plate arcs, and crystals like the lower one made the circular halos. Fairbanks, November 15, 2003. Σ = 4°.
FIGURE 15.11 Lunar display with odd radius plate arcs. The simulation was made using plate oriented crystals like the one shown. The yellow and red dots in the photo are 32° and 34.9° from the moon; like the yellow and red dots in Figure 8.3, they argue in favor of the Steinmetz and Weickmann value $x = 28°$ over the Besson and Humphreys value $x = 25°$; see the discussion on page 121. The asymmetry of the halos is not real but results from the photo not being centered on the moon. Fairbanks, 4:19 a.m., January 6, 2004. Moon elevation 35°.
The simulation is a fairly good match for the photo, but it is not perfect. The 9° plate arc in the simulation is too strong, and the 24° plate arcs do not seem to be shaped quite right. This is fairly typical of our simulations of odd radius halo displays. That is, the simulations tend to be good, but not perfect. It makes us a bit nervous.

The display in Figure 15.11 has 9°, 18°, 20°, 23°, 24°, and 35° plate arcs. Several stars are identified in the photo, and their angular distances from the moon are given. The location of the star TYC 3391-2638-1 shows that the arc at the top of the photo is indeed the upper 23° plate arc rather than the upper tangent arc; the arc is too far from the moon to be the tangent arc. And the location of α Ori shows that the concentration of light at the bottom of the photograph is indeed the lower 20° plate arc rather than the lower tangent arc.

There are no stars visible near the 35° plate arc to confirm its angular distance from the moon. We have therefore positioned the red dot so that its angular distance from the moon is 34.9°—precisely the theoretical radius of the 35° halo given in Table 8.1. Although the 35° arc here is a plate arc rather than a circular halo, the tilts of the crystals are large enough so that the 35° arc would have been in contact with the circular halo had it been present. Hence, since the red dot is on the inner edge of the arc, this display gives good empirical support for the theoretical value $\Delta_{\text{min}} = 34.9°$ for the 35° halo radius.

For comparison, a yellow dot is positioned 32° from the moon. To understand the significance of this second dot, recall from Chapter 11 that historically the correct value for $x$ has been in doubt, with $x = 28°$ (Steinmetz and Weickmann) and $x \approx 25°$ (Besson and Humphreys) being two competing values for $x$. For the halo under consideration here, the value $x = 28°$ gives the theoretical value $\Delta_{\text{min}} = 34.9°$ of the preceding paragraph, whereas $x = 25°$ gives $\Delta_{\text{min}} = 31.9°$. Thus the location of the halo with respect to the two dots argues strongly for the Steinmetz and Weickmann value $x = 28°$. We came to the same conclusion on page 121 in connection with Figure 8.3.

Halo associations

Whereas in Figure 15.8 the odd radius plate arcs were the 18° and 23° arcs, in Figures 15.12 and 15.13 they are mainly the 9° and 24° arcs. These halo associations are common; the 18° and 23° arcs tend to occur together, and the 9° and 24° arcs tend to occur together. These associations are not limited to plate arcs; Figure 8.4, for example, illustrates the 18°–23° association for circular halos.

You can begin to understand halo associations by comparing the pyramidal crystal in Figure 15.8 with the pyramidal crystals in Figures 15.12 and 15.13.
FIGURE 15.12  Halo display with upper 9° plate arc—the bright spot above the sun. Also visible are bright 22° plate arcs at left and right (i.e., ordinary parhelia), a faint 22° circular halo, and faint upper 24° plate arcs at 2:00 and 10:00 just outside the 22° halo. Crystals like the lower one made the plate arcs in the simulation, and crystals like the upper one made the 22° halo. Fairbanks, February 26, 2000. Σ = 7°.
CHAPTER 15 • ODD RADIUS PLATE ARCS

FIGURE 15.13  Lunar display with lower 9° and lower 24° plate arcs. The lower 9° plate arc is the bright spot below the moon, and the lower 24° plate arcs are the areas of illumination at 4:00 and 8:00 just outside the 22° circular halo. Crystals like the lower one made the plate arcs in the simulation, and crystals like the upper one made the circular halos. Fairbanks, 6:04 P.M., January 5, 2004. Moon elevation 25°.
Prism faces are necessary for the ray paths for the 9° and 24° arcs, but large prism faces interfere with the ray paths for the 18° and 23° arcs. There is more to it than that, however, and the halo associations do not invariably materialize.

**The 23° plate arc**

The lovely halo in Figure 15.14 was photographed by Timo Viinanen in Finland in August of 2004. Viinanen did well to recognize that the display was special and then to get a sequence of photos documenting its evolution over the course of an hour and a half. The bright halo shown here was by far the most prominent halo of the display, but faint odd radius halos were also present from time to time. The most conspicuous of the faint halos seem to have been 18° plate arcs.

We believe that the halo in the photo is the upper 23° plate arc. The identification is not automatic, however, since theoretically the upper 23° plate arc barely differs in appearance from the upper suncave Parry arc. The two arcs are shown in the simulations in Figure 15.15. It is not surprising that the arcs look alike, since they have nearly the same wedge angle and nearly the same pole.

What, then, are we seeing in the photograph—a Parry arc or an odd radius plate arc? Halo displays that contain unequivocal Parry arcs almost always contain well developed column arcs as well, as in Figures 1.8, 6.7, and 6.8, so the absence of column arcs in the photo favors the odd radius plate arc identification. So does the presence of the 18° plate arcs in the display.

That is about as far as we can go, without more information. For the sun elevation in question the Parry arc is theoretically about a degree closer to the sun than is the plate arc. If the sun had been in the photo, then careful measurements might have distinguished between the two possibilities. And had polarization observations been made during the display, as described briefly in Appendix C and in much more detail by Können [40], they might also have been decisive.

Both explanations—Parry arc and odd radius plate arc—require some rationalizing regarding the halo associations predicted by the simulations. According to the simulations, either the lower 20° plate arc or the lower suncave Parry arc should probably have been visible, though faint. Viinanen did not see them, so we have to believe that the halo-making cloud did not extend into the relevant region, or that the cloud was too thick there to make a halo. (The other, unlabeled halos in the simulations would perhaps have been too faint to be seen in reality.)

Figure 15.16 is a similar display, but with $\Sigma = 22°$. The photographer, Les Cowley, analyzed the display and concluded that this halo, too, was probably the upper 23° plate arc. Some of his analysis can be found on his website; see the footnote on page 51.
The simulations were made with the indicated crystals—plate oriented pyramids for the left, and Parry oriented columns for the right. The simulations include much more sky than in the photograph above. $\Sigma = 41^\circ$. 

**FIGURE 15.15** Comparison of upper 23° plate arc (left) and upper suncave Parry arc (right). The simulations were made with the indicated crystals—plate oriented pyramids for the left, and Parry oriented columns for the right. The simulations include much more sky than in the photograph above. $\Sigma = 41^\circ$. 

**FIGURE 15.14** Probable upper 23° plate arc. Hämeenlinna, Finland, August 8, 2004. $\Sigma = 41^\circ$. Photo © Timo Viinanen.
The remarkable brightness and quality of the halos in these two photographs raises the question of whether we might be missing the boat entirely. Could these halos be instances of some new halo altogether—neither Parry arc nor 23° plate arc? We do not think so, but we can’t be positive.

More on the upper 23° plate arc can be found in an article by Riikonen et al [61]. They believe that upper 23° plate arcs are more common than has been thought, and that many of them have been mistaken for upper tangent arcs. It is easy to see how this could happen, especially if the arc were of only mediocre quality, like the one in Figure 15.4, for example.

An open problem

We do not understand the two very similar displays shown in Figures 15.17 and 15.18. In each photo the two bright arcs closest to the sun look just right for the 18° plate arcs 13 25 and 13 27, and we can easily simulate them. In particular, it is quite all right that the arcs are farther than $\Delta_{\text{min}} = 18.3°$ (orange dots) from the sun, since the sun elevation here is not close to the theoretical sun elevation 31° needed for contact, and since the arcs show at least a hint of sunvex shape,
FIGURE 15.17  Odd radius plate arcs? The bright inner arcs are probably the 18° plate arcs, but the fainter arcs just outside them are harder to identify. See text. The orange and green dots are 18.3° and 21.8° from the sun, the Δ_min-values for the 18° and 22° halos. Fairbanks, January 25, 1996. Σ = 6°. 20mm lens.

FIGURE 15.18  Similar. Fairbanks, February 2, 1996. Σ = 6°. 28mm lens.
suggesting that the tilts of the crystals are not large. But the identity of the two outer arcs is in doubt. They might be the 22° plate arcs, they might be the 18° plate arcs 23 15 and 23 17, or they might be neither. We are unable to make a simulation that is completely satisfactory. Is our failure to simulate the display a consequence only of a missing detail, or does it represent some fundamental misconception? We do not know. The fact that such similar displays occurred on two different days suggests that the circumstances were not especially pathological and that a simulation should not require a lot of tweaking of crystal shapes and orientations. But we have had no luck.

**Finding the halo poles**

Using the method of Chapter 14, you can verify the pole locations shown in Figure 15.7 for yourself, if you are willing to build a model of a standard pyramidal crystal (Appendix A). Since the crystal orientations are plate orientations, then the spin vector $\mathbf{P}$ is $\mathbf{N}_1$, the normal vector to face 1. If you want the pole of the plate arc due to, say, wedge 13 27, you just put wedge 13 27 in standard orientation; the vector $\mathbf{P} = \mathbf{N}_1$ then points in the direction of the pole. This is illustrated in Figure 15.19, but the figure is a poor substitute for a crystal model. With the model in your hands, you will easily see whether the relevant wedge is in standard orientation. This is not so clear from the figure.

The remaining pole locations can be verified in the same way, with the crystal model. Remember that this is more than amusement. By finding the poles, you are finding all (refraction) plate arcs that result from standard pyramidal crystals.

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2 Their poles are on the rear hemisphere and not seen in Figure 15.7.
A look at the crystals

This chapter concludes with Figures 15.20 and 15.21, showing an odd radius halo display and some of the crystals that made it. Many of the crystals were pyramidal, as expected.

Perhaps it is a good time to recall that pyramidal ice crystals are barely known in the scientific literature and that, where known, they are often regarded as impossibly rare. But every odd radius halo display, like the one in Figure 15.20, betrays the presence of countless such crystals. At such moments pyramidal ice crystals are far from rare.

FIGURE 15.20  Odd radius lunar display. Some of the crystals that fell during the display are shown in Figure 15.21. Fairbanks, December 24, 2004. $\Sigma = 39^\circ$. 
FIGURE 15.21 ESEM photos of some crystals collected during the display of Figure 15.20. The black and white scale bars on the photos, clockwise from upper left, are 30, 50, 50, 45, 45, and 100μ. The selection of photos exaggerates the frequency of pyramidal crystals in the crystal sample, but not by much. Photos © Kenneth P. Severin.